In The Name of God, The Compassionate, The Merciful

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Statistical Pattern Recognition (CE-725)
Department of Computer Engineering
Quiz #2 solution (Mathematical Review)
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1. **(10 points)** Consider two Gaussians $N_1(\mu_1, \Sigma_1)$ and $N_2(\mu_2, \Sigma_2)$ with the following parameters:

$$\mu_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \mu_2 = \begin{bmatrix} c \\ 0 \end{bmatrix}, \qquad \Sigma_1 = \begin{bmatrix} p & 0 \\ 0 & q \end{bmatrix},\ \Sigma_2 = \begin{bmatrix} q & 0 \\ 0 & p \end{bmatrix}$$

Each point in the 2-d Cartesian space belongs to one of these Gaussians (A point x belongs to N_1, if $P_1(x)>P_2(x)$ and vice versa, where P_1 and P_2 are probability distribution functions of N_1 and N_2). Itmeans that these two Gaussians partition the whole space to two subspaces. Write down the boundary equation in the following form:

$$A\begin{bmatrix} x_1^2 \\ x_2^2 \end{bmatrix} + B\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + C = 0$$

Solution: We can obtain the boundary equation by solving $P_1(x) = P_2(x)$, then we have:

$$P_1(x) = P_2(x)$$

$$\frac{1}{2\pi\sqrt{|\Sigma_1|}} e^{-\frac{1}{2}(x-\mu_1)^t \Sigma_1^{-1}(x-\mu_1)} = \frac{1}{2\pi\sqrt{|\Sigma_2|}} e^{-\frac{1}{2}(x-\mu_2)^t \Sigma_2^{-1}(x-\mu_2)}$$

$$\frac{1}{2\pi\sqrt{pq}} e^{-\frac{1}{2}(x-\mu_1)^t \Sigma_1^{-1}(x-\mu_1)} = \frac{1}{2\pi\sqrt{qp}} e^{-\frac{1}{2}(x-\mu_2)^t \Sigma_2^{-1}(x-\mu_2)}$$

$$-\frac{1}{2}(x-\begin{bmatrix} 0 \\ 0 \end{bmatrix})^t \begin{bmatrix} p^{-1} & 0 \\ 0 & q^{-1} \end{bmatrix}(x-\begin{bmatrix} 0 \\ 0 \end{bmatrix}) = -\frac{1}{2}(x-\begin{bmatrix} c \\ 0 \end{bmatrix})^t \begin{bmatrix} q^{-1} & 0 \\ 0 & q^{-1} \end{bmatrix}(x-\begin{bmatrix} c \\ 0 \end{bmatrix})$$

We can simplify above equations as follow:

$$x_1^2(q-p) + x_2^2(p-q) + 2pcx_1 - pc^2 = 0$$

$$[(q-p) \quad (p-q)]\begin{bmatrix} x_1^2 \\ x_2^2 \end{bmatrix} + [2pc \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - pc^2 = 0$$