

Name:

Student ID#:

Statistical Pattern Recognition (CE-725)
Department of Computer Engineering
Quiz #4 Solution - Spring 2010

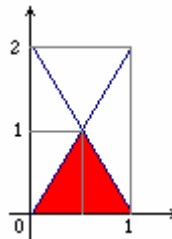
Consider the following 2-class classification problem involving a single feature x . Assume equal class priors.

$$p(x|w_1) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad p(x|w_2) = \begin{cases} 2-2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

a. (4 points) State the Bayes decision rule.

$$\begin{aligned} \frac{c_1}{c_2} p(w_1|x) &\geq p(w_2|x) \\ \frac{1}{2} p(x|w_1) &\geq \frac{1}{2} p(x|w_2) \\ x &\geq 1-x \\ x &\geq \frac{1}{2} \end{aligned}$$

b. (3 points) What is the Bayes classification error (hint: sketch the densities and boundary)?



$$\text{Error} = \frac{\text{area of red triangle}}{\text{total area}} = \frac{1}{2} \left(\frac{1}{2} \right) = 0.25$$

c. (3 points) How will the decision boundary change if the prior for class w_1 is increased to 0.7?

$$\begin{aligned} \frac{c_1}{c_2} p(w_1|x) &\geq p(w_2|x) \\ 0.7 * p(x|w_1) &\geq 0.3 * p(x|w_2) \\ x &\geq \frac{3}{10} \end{aligned}$$