In The Name of God, The Compassionate, The Merciful

Name: ....

Student ID#: ....

Statistical Pattern Recognition (CE-725)
Department of Computer Engineering
Quiz #6 (Mini Exam) solutions - Spring 2010

1. (40 points) Assume a two-class (w_1 and w_2) classification problem with the Gaussian densities and the following parameters:

Prior probabilities: $P(w_1) = P(w_2)$

Means: $\mu_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mu_2 = \begin{bmatrix} d \\ e \end{bmatrix}$

Covariances: $\Sigma_1 = \begin{bmatrix} a & c \\ c & b \end{bmatrix}, \ \Sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

where $ab-c^2 = 1$.

a) Find the Bayesian discriminant function and the boundary equation (write them in the simplest form).

Sol: The discriminant function is:

$$g_i(x) = \frac{1}{2}(x-\mu_i)^t \Sigma_i^{-1}(x-\mu_i) - \frac{1}{2}\ln|\Sigma_i|$$

The decision boundary is given by $g_1(x) = g_2(x)$, then we have:

$$(b-1)x_1^2 + (a-1)x_2^2 - 2cx_1x_2 + 2dx_1 + 2ex_2 - d^2 - e^2 = 0$$

b) Determine the constraints on the values of a, b, c, d and e, such that the resulting discriminant function results in a linear decision boundary.

Sol: From the boundary equation or from the $\Sigma_1 = \Sigma_2$ we'll have a=b=1 and c=0.

c) Let a=2, b=1, c=0, d=4, e=4. Find the Fisher's projection direction (w). Suppose that the number of class 1 and class 2 samples are equal.

Sol: The Fisher's projection direction is $w = S_w^{-1}(\mu_1 - \mu_2)$, then:

$$S_w = S_1 + S_2 = n\Sigma_1 + n\Sigma_2 = n\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} + n\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = n\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow S_w^{-1} = \frac{1}{n}\begin{bmatrix} 1/3 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$w = S_w^{-1}(\mu_1 - \mu_2) = \frac{1}{n}\begin{bmatrix} 1/3 & 0 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} -4 \\ -4 \end{bmatrix} = \begin{bmatrix} -4/3n \\ -4/2n \end{bmatrix}$$

Then the Fisher's projection direction is [-1/3 -1/2]^T.

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2. (20 points) What is cross-validation? Give examples of cross-validation methods.

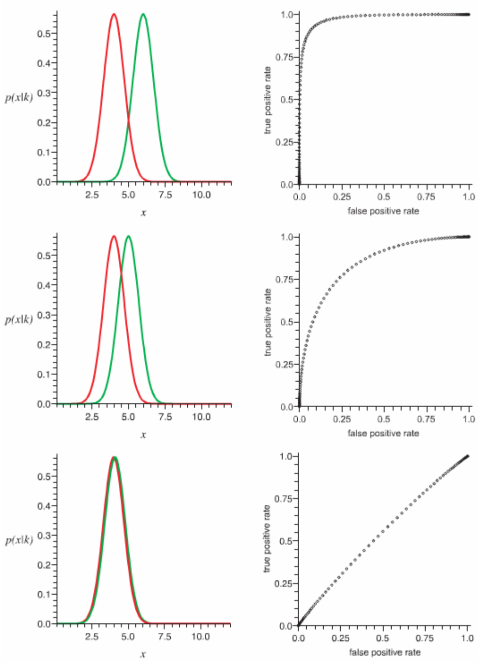
> **Sol:** Cross-validation amounts to splitting the training data multiple times into training set and validation set. Training is performed on the training set and test on the validation set. The validation results are then averaged.
>
> Note that this does not preclude of reserving a separate test set for the final testing.
>
> Examples of cross-validation techniques include k-fold cross-validation, bootstrap, leave-one out.

3. (20 points) Assume a one-dimensional two-class problem with Gaussian densities with the same variances and means such that $\mu_1 < \mu_2$.

The decision boundary of these classes will be of the form $x = \theta$, which θ is in the range of $(-\infty, +\infty)$. A ROC curve can be drawn based on θ.

If μ_2 become smaller (closer to μ_1), how the ROC curve will be changed?

> **Sol:** The ROC curve will be closed to the 45 degree diagonal of the ROC space.



4. (20 points) Given the following set of prototypes:

S1: (0,1), (0,2)

S2: (1,0), (2,0)

Apply pseudo-inverse procedure to find a solution vector for a linear discriminant function.

Hint: if $A = \begin{bmatrix} 5 & 0 & 3 \\ 0 & 5 & 3 \\ 3 & 3 & 4 \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} 1.1 & 0.9 & -1.5 \\ 0.9 & 1.1 & 1.5 \\ -1.5 & -1.5 & 2.5 \end{bmatrix}$

> **Sol:** The pseudo-inverse weights vector obtains from $w = (X^T X)^{-1} X^T b$, then:

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$$X=\begin{bmatrix}0&1&1\\0&2&1\\1&0&1\\2&0&1\end{bmatrix},\ b=\begin{bmatrix}1\\1\\-1\\-1\end{bmatrix}\Rightarrow\ X^tX=\begin{bmatrix}5&0&3\\0&5&3\\3&3&4\end{bmatrix},X^tb=\begin{bmatrix}-3\\3\\0\end{bmatrix}$$

$$\Rightarrow\ w=(X^TX)^{-1}X^tb=\begin{bmatrix}-0.6\\0.6\\0\end{bmatrix}$$

Hence, the discriminat boundary will be as $x_1=x_2$;