In The Name of God, The Compassionate, The Merciful

Name: .....

Student ID#: .....

Statistical Pattern Recognition (CE-725)
Department of Computer Engineering
Quiz #6 (Mini Exam) - Spring 2010

1. (40 points) Assume a two-class (w_1 and w_2) classification problem with the Gaussian densities and the following parameters:

Prior probabilities: $P(w_1) = P(w_2)$

Means: $\mu_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mu_2 = \begin{bmatrix} d \\ e \end{bmatrix}$

Covariances: $\Sigma_1 = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$, $\Sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

where $ab-c^2 = 1$.

a) Find the Bayesian discriminant function and the boundary equation (write them in the simplest form).

b) Determine the constraints on the values of a, b, c, d and e, such that the resulting discriminant function results in a linear decision boundary.

c) Let a=2, b=1, c=0, d=4, e=4. Find the Fisher's projection direction (w). Suppose that the number of class 1 and class 2 samples are equal.

2. (20 points) What is cross-validation? Give examples of cross-validation methods.

3. (20 points) Assume a one-dimensional two-class problem with Gaussian densities with the same variances and means such that $\mu_1 < \mu_2$.

The decision boundary of these classes will be of the form $x = \theta$, which θ is in the range of $(-\infty,+\infty)$. A ROC curve can be drawn based on θ.

If μ_2 become smaller (closer to μ_1), how the ROC curve will be changed?

4. (20 points) Given the following set of prototypes:

S1: (0,1), (0,2)

S2: (1,0), (2,0)

Apply pseudo-inverse procedure to find a solution vector for a linear discriminant function.

Hint: if $A = \begin{bmatrix} 5 & 0 & 3 \\ 0 & 5 & 3 \\ 3 & 3 & 4 \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} 1.1 & 0.9 & -1.5 \\ 0.9 & 1.1 & 1.5 \\ -1.5 & -1.5 & 2.5 \end{bmatrix}$