In The Name of God, The Compassionate, The Merciful

Name: ....

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Statistical Pattern Recognition (CE-725)
Department of Computer Engineering
Quiz#8 solution - Spring 2010

a. (10 points) Prove that $K(x,x') = (x^Tx' + 1)^2$ is a valid kernel.

b. (10 points) Find the corresponding transformation function of this kernel.

(Hint : You may guess this function instead of using Mercer's Theorem)

Sol:

a.

$$\int_{R^{2n}} (x^T x'+1)^2 f(x)f(x')dxdx' = \int_{R^{2n}} \left(\sum_{i=1}^{n} x_i x_i' + 1\right)^2 f(x)f(x')dxdx'$$

$$= \sum_{i=1}^{n} \int_{R^{2n}} (x_i x_i')^2 f(x)f(x')dxdx' +$$

$$2\sum_{i,j=1}^{n} \int_{R^{2n}} (x_i x_i')(x_j x_j') f(x)f(x')dxdx' +$$

$$2\sum_{i=1}^{n} \int_{R^{2n}} (x_i x_i') f(x)f(x')dxdx' +$$

$$\int_{R^{2n}} f(x)f(x')dxdx'$$

b. We have $\left(\sum_{i=1}^{n} x_i x_i' + 1\right)^2 = \sum_{i=1}^{n} (x_i x_i')^2 + 2\sum_{i,j=1}^{n} (x_i x_i')(x_j x_j') + 2\sum_{i=1}^{n} (x_i x_i') + 1$

Therefore we can guess that

$$\phi = (x_1^2, \ldots, x_n^2, \sqrt{2}x_1x_2, \sqrt{2}x_1x_3, \ldots, \sqrt{2}x_nx_{n-1}, \sqrt{2}x_1, \ldots, \sqrt{2}x_n, 1)$$