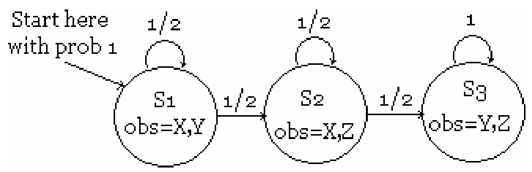
In The Name of God, The Compassionate, The Merciful

Name: ....

Student ID#: ....

Statistical Pattern Recognition (CE-725)
Department of Computer Engineering
Quiz #9 - Spring 2010

Consider the following HMM:



$$a = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \end{bmatrix},\ b(X) = \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix},\ b(Y) = \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix},\ b(Z) = \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix},\ \pi = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Where

$$a_{ij} = P(q_{t+1} = S_j \mid q_t = S_i),\ b_i(k) = P(O_t = k \mid q_t = S_i)$$

Suppose we have observed this sequence: XZXYYZYZZ

(In long-hand: $O_1 = X, O_2 = Z, O_3 = X, O_4 = Y, O_5 = Y, O_6 = Z, O_7 = Y, O_8 = Z, O_9 = Z$).

Fill in the following table with $\alpha_i(t)$ values, remembering the definition:

$$\alpha_t(i) = P(O_1, O_2, \ldots O_t, q_t = s_i)$$

So for example, $\alpha_3(2) = P(O_1 = X, O_2 = Z, O_3 = X, q_3 = S_2)$.

t	$\alpha_t(1)$	$\alpha_t(2)$	$\alpha_t(3)$
1			
2			
3			
4			
5			
6			
7			
8			
9			

Warning: this is a question that will take a few minutes if you really understand HMMs, but could take hours if you don't!

Sol:

t	$\alpha_t(1)$	$\alpha_t(2)$	$\alpha_t(3)$
1	2^{-1}		
2		2^{-3}	
3		2^{-5}	
4			2^{-7}
5			2^{-8}
6			2^{-9}
7			2^{-10}
8			2^{-11}
9			2^{-12}