In The Name of God, The Compassionate, The Merciful

Name: ....

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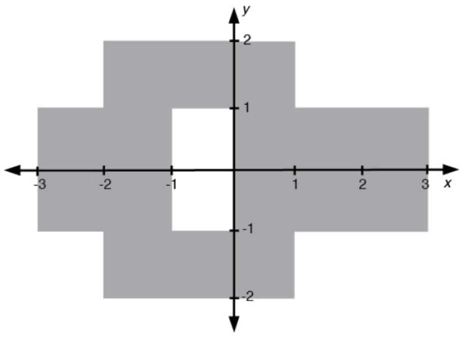
Statistical Pattern Recognition (CE-725)
Department of Computer Engineering
Quiz #4 Solution (Probabilistic Classification) - Spring 2011

1. Suppose x and y are random variables. Their joint density, depicted below, is constant in the shaded area and 0 elsewhere.



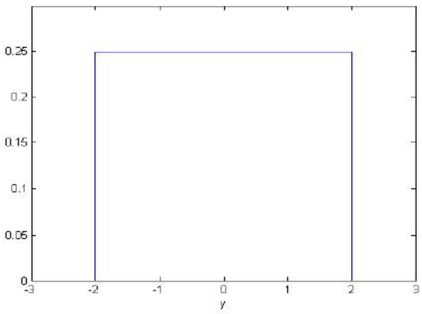
a. (2 points) Let w_1 be the case when x<0, and w_2 be the case when x>0. Determine the a priori probabilities of the two classes $P(w_1)$ and $P(w_2)$.

By simply counting the number of unit squares in the shaded areas on the left and right sides of the line x=0; we can directly and out that there are 8 unit squares on each side. Thus, the two cases are equally likely, i.e. P(w1) = P(w2) = 0.5.

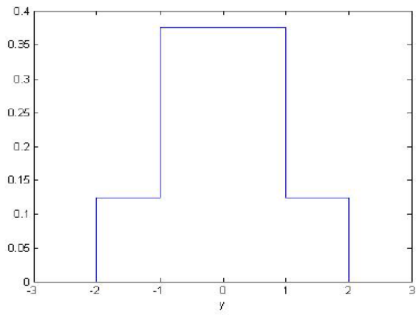
b. (2 Points) Let y be the observation from which we infer whether w_1 or w_2 happens. Find and sketch the likelihood functions, namely, the two conditional distributions $p(y|w_1)$ and $p(y|w_2)$.

It's also pretty straightforward to obtain the likelihood functions p(y|w1) and p(y|w2) by counting the number of unit squares for different ranges of y. We just need to be careful with normalizing them such that the integral of the distribution 1. The sketches are as follows:

P(y|w1)



P(y|w2)



c. (6 points) Find the posterior decision rule, and calculate the probability of error. Please note that there will be ambiguities at decision boundaries, but how you classify when y falls on the decision boundary doesn't affect the probability of error.

Since the prior probabilities are equal, the posterior decision rule simply relies on the comparison of the two likelihood function, In other words, it becomes a ML decision rule. The decision rule can be summarized as

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$$\hat{w}=\begin{cases} w_1 & \text{if } -2<y\leq -1 \text{ or } 1<y\leq 2 \\ w_2 & \text{if } -1<y\leq 1 \end{cases}$$

Hence the probability of error is

$$\begin{aligned} P(e) &= P(\hat{w}=w_2|w_1)P(w_1)+P(\hat{w}=w_1|w_2)P(w_2) \\ &= \frac{1}{2}P(-1<y\leq 1|w_1)+\frac{1}{2}P(-2<y\leq -1,1<y\leq 2|w_2) \\ &= \frac{1}{2}*\frac{1}{2}+\frac{1}{2}*\frac{1}{4}=\frac{3}{8} \end{aligned}$$