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# Statistical Pattern Recognition (CE-725)
# Department of Computer Engineering
# Quiz #5 solutions (Statistical Classification) - Spring 2011

**1.** **(3 points)** Consider a decision problem using one-dimensional feature vectors. Find the Likelihood Ratio Test for the following two class conditional densities:

$$P(x|w_1)=\frac{1}{2}e^{-\frac{x}{2}},x>0 \qquad P(x|w_2)=\frac{1}{4}xe^{-\frac{x}{2}},x>0$$

Hint: You must calculate the $P(x|w_1)/P(x|w_2)$ ratio.

**Sol:**

$$\frac{P(x|w_1)}{P(x|w_2)}=\frac{\frac{1}{2}e^{-\frac{x}{2}}}{\frac{1}{4}xe^{-\frac{x}{2}}}=\frac{2}{x}$$

The decision rule is to choose $w_1$ if $2/x >= T$, and choose $w_2$ otherwise. And the decision boundary is $x=2/T$.

**2.** For the above problem, find the thresholds of likelihood ratio for the following cases:

**a. (2 points)** Bayesian minimum risk, with $\lambda 11= \lambda 22=0$, $\lambda 12=1$, $\lambda 21=2$, $P(w_1) =2/3$, and $P(w_2) =1/3$.

Hint: For example, the threshold of likelihood ratio for Class Conditional Method is T=1, or the threshold of likelihood ratio for MAP Method is $T=P(w_2)/P(w_1)$.

**Sol:**

$$T=\frac{(\lambda_{12}-\lambda_{22})P(w_2)}{(\lambda_{21}-\lambda_{11})P(w_1)}=\frac{1}{4}$$

**b. (5 points)** Neyman-Pearson with $P_{FP}= 0.1$.

**Sol:**

The decision boundary is $2/x = T$, or $x=2/T$, then we have:

$$P_{FA}=0.1=\int_{-\infty}^{2/T}\frac{1}{4}xe^{-x/2}dx=\ldots \Rightarrow T=\ldots$$