In The Name of God, The Merciful, The Beneficent

Pattern Recognition – CE725
Department of Computer Engineering
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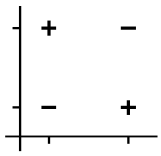
Classification and Discriminant Functions

1. Consider the XOR problem, in which there are two input attributes x and y which take on the values 0 and 1. The output class is positive if and only if $x \neq y$.



What will happen if you try to train a Gaussian-based Bayes Classifier on such a dataset? Assume that the classifier is able to learn arbitrary covariance matrices.

2. We consider a classification problem in dimension d=2, with k=3 classes, where: $p(x \mid w_i) \sim N(\mu_i, \Sigma_i),\ i = 1,2,3$, with

$$\mu_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix},\ \mu_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix},\ \mu_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ \Sigma_i = \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1/3 \end{bmatrix}$$

a. Calculate the discriminant function $g_i(x)$ for each class.

b. Express your discriminant functions in the form of linear discriminant functions.

c. determine and plot the decision boundaries.

3. Consider the following 2-class classification problem involving a single feature x. Assume equal class priors and 0-1 loss function.

$$p(x \mid w_1) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & otherwise \end{cases} \qquad p(x \mid w_2) = \begin{cases} 2-2x & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

a. sketch the two densities.

b. state the Bayes decision rule and show the decision boundary.

c. what is the Bayes classification error?

d. How will the decision boundary change if the prior for class w_1 is increased to 0.7?

Modeling Parameter Estimation

4. What are the sources of error when modeling the samples?

5. Let $x_1, x_2, ..., x_n$ be i.i.d. samples that follow the following pdf:

$$p(x \mid \theta) = \begin{cases} \theta^2 x e^{-\theta x} & x \ge 0 \\ 0 & otherwise \end{cases}$$

Find the MLE of θ.

PCA and FLD

6. when the results of data projection on Fisher direction and data projection on first component of PCA are the same?

7. Assume we have 100 samples equally in two Classes (A & B) in 2D space with following means and covariances:

$$\mu_A = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \Sigma_A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \qquad \mu_B = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad \Sigma_B = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$$

a. Find the first principal component$u$.
b. Use Fisher's method to find the best feature direction w.