Name: ....

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Statistical Pattern Recognition (CE-725)
Department of Computer Engineering
Mini Exam #1 solutions (SVM & Kernels) - Spring 2011

1. a) (20 extra points) Prove that if we remove non-support vector data from the data set the solution of the SVM will remain unchanged.

b) (20 points) Consider the standard two-class SVM with the hinge loss. Argue that under a given value of C,

$$\text{Leave\_One\_Out error} \le \frac{\#\text{SVs}}{l},$$

where l is the size of training data and #SVs is the number of support vectors obtained by training SVM on the entire set of training data. Hint: use part a.

Sol: Since the decision function only depends on the support vectors, removing a non-support vector from the training data and them re-training an SVM would lead to the same decision function. Also, non-support vectors must be classified correctly. As a result, errors found in the leave-one-out validation must be caused by removing the support vectors, proving the desires result.

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2. (30points) Assume that the embedding spaces of the kernels k_1 and k_2 are represented as φ_1 and φ_2, respectively. Find the embedding spaces of these kernels:

a) $k = k_1 + k_2$

Sol: $\varphi = (\varphi_1, \varphi_2)$

b) $k = k_1 . k_2$

Sol: $\varphi = \left(\varphi_{11}\varphi_{21}, \varphi_{11}\varphi_{22}, \ldots, \varphi_{11}\varphi_{2m}, \varphi_{12}\varphi_{21}, \ldots, \varphi_{1n}\varphi_{2m}\right)$

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3. (50 points) Suppose we have six training points from two classes. We have four points from class 1: (0.2, 0.4), (0.4, 0.8), (0.4, 0.2), (0.8, 0.4) and two points from class 2: (0.4, 0.4), (0.8, 0.8).

Unfortunately, the points cannot be separated by a linear classifier. The kernel trick is to find a mapping of x to some feature vector $\phi(x)$ such that there is a function K called kernel which satisfies $K(x,x') = \phi(x)^T \phi(x')$. And we expect the points of $\phi(x)$ to be linearly separable in the feature space. Here, we consider the following normalized kernel:

$$k(x,x') = \frac{x^T x'}{\|x^T\| \|x'\|}$$

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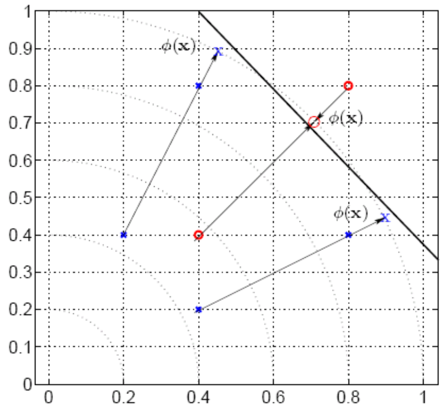
a) What is the feature vector $\phi(x)$ corresponding to this kernel? Draw $\phi(x)$ for each training point x, and specify from which point it is mapped.

Sol: $\phi(x) = \frac{x}{\|x\|}$



b) Now the feature vectors are linearly separable in the feature space. The maximum-margin decision boundary in the feature space will be a line in $\mathbb{R}^2$, which can be written as $w_1 x + w_2 y + c = 0$. What are the values of the coefficients w_1 and w_2? (Hint: you don't need to compute them.)

Sol: (w1,w2) = (1, 1)

c) Specify the support vectors.

Sol: All po ints are SV.

d) Draw the decision boundary in the original input space resulting from the normalized linear kernel. Briefly explain your answer.