

In the Name of God, The Compassionate, The Merciful

Name :

Student ID# :

**Statistical Pattern Recognition
Department of Computer Engineering
Mini Exam #2 (Graphical Models) – Spring 2011**

Q1. [60 Pts.] Assume that X_1, \dots, X_n, \dots has the Markov property with X_i takes value in $\{0, 1\}$. We have:

$P(X_i = 1 X_{i-1} = 1)$	0.5
$P(X_i = 0 X_{i-1} = 0)$	0.7

We know that $P(X_1 = 0) = 0.5$. Further assume that we have another sequence of random variables Z_1, \dots, Z_n, \dots such that each Z_i takes its value in $\{0, 1\}$ and is conditionally independent of all other variables given X_i . We have :

$P(Z_i = 1 X_i = 0)$	1
$P(Z_i = 0 X_i = 1)$	0.4

Find $P(Z_3 = 0 | Z_1 = 0)$.

Solution:

$$P(Z_3 = 0 | Z_1 = 0) = P(Z_3 = 0, Z_2 = 0 | Z_1 = 0) + P(Z_3 = 0, Z_2 = 1 | Z_1 = 0) \\ = P(000)/P(0) + P(010)/P(0)$$

We may use forward algorithm to find the value of $\delta_i(\mathbf{s})$ for $\text{seq} = 010$:

$0.5 * 0 = 0$	$0 + 0.2 * 0.5 * 1 = 0.1$	0
$0.5 * 0.4 = 0.2$	$0 + 0.2 * 0.5 * 0.6 = 0.06$	$(0.1 * 0.3 + 0.06 * 0.5) * 0.4 = 0.024$

Further, for $\text{seq} = 000$, we have

$0.5 * 0 = 0$	$0 + 0.2 * 0.5 * 0 = 0$	$0 + 0.04 * 0.5 * 0 = 0$
$0.5 * 0.4 = 0.2$	$0 + 0.2 * 0.5 * 0.4 = 0.04$	$0 + 0.04 * 0.5 * 0.4 = 0.008$

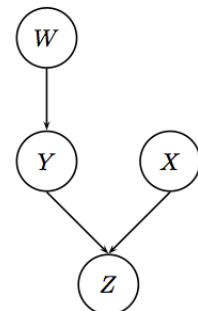
In addition $P(0) = P(0|X_1 = 0)P(X_1 = 0) + P(0|X_1 = 1)P(X_1 = 1) = 0 + 0.4 * 0.5 = 0.2$.

Therefore, we have : $P(Z_3 = 0 | Z_1 = 0) = 0.024/0.2 + 0.008/0.2 = 0.12 + 0.04 = 0.16$.

Q2. [40 Pts.] Consider the following Bayesian network with each variable taking value in $\{0, 1\}$:

Find the following probabilities based on the conditional probabilities of each node given all its parents.

- a) $P(Z=1 | W=1, X=1)$
- b) $P(W=1 | X=1, Y=1, Z=1)$



Solution:

$$a) P(Z=1 | W=1, X=1) = P(Z=1|W=1, X=1, Y=1) P(Y=1 | W=1, X=1) \\ + P(Z=1|W=1, X=1, Y=0) P(Y=0 | W=1, X=1) \\ = P(Z=1| Y=1, X=1) P(Y=1|W=1) +$$

$$\begin{aligned}
& P(Z=1|Y=0, X=1) P(Y=0|W=1). \\
\text{b) } P(W=1 | X=1, Y=1, Z=1) &= \frac{P(X=1, Y=1, Z=1, W=1)}{P(X=1, Y=1, Z=1, W=1) + P(X=1, Y=1, Z=1, W=0)} \\
&= \frac{P(X=1) * P(W=1) * P(Y=1|W=1) * P(Z=1|Y=1, X=1)}{P(X=1) * P(W=1) * P(Y=1|W=1) * P(Z=1|Y=1, X=1) + P(X=1) * P(W=1) * P(Y=0|W=1) * P(Z=1|Y=0, X=1)} \\
&= \frac{P(Y=1|W=1) * P(Z=1|Y=1, X=1)}{P(Y=1|W=1) * P(Z=1|Y=1, X=1) + P(Y=0|W=1) * P(Z=1|Y=0, X=1)}
\end{aligned}$$