In The Name of God, The Compassionate, The Merciful

Name: .....

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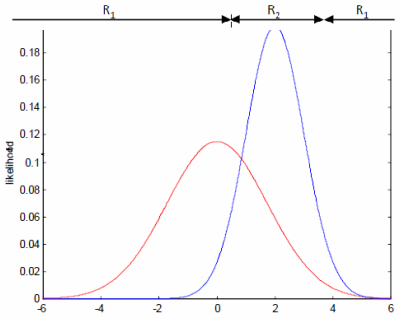
Statistical Pattern Recognition (CE-725)
Department of Computer Engineering
Quiz #5 (Classification – Probabilistic Methods) - Spring 2012

1. (50 points) Consider a binary classification problem with likelihoods $P(x|C_1)\sim N(0, \sqrt{3})$ and $P(x|C_2)\sim N(2, 1)$. Assume $P(C_1)=P(C_2)$, $\lambda_{ii}=0$, $\lambda_{12}=1$ and $\lambda_{21}=\sqrt{3}$. Find the decision boundary which minimizes P(error). Hint: $\sqrt{3}\approx 1.73$



Sol:

$$\frac{N(0,\sqrt{3})}{N(2,1)}=\frac{1}{\sqrt{3}}\Rightarrow\frac{x^2}{3}-(x-2)^2=0\Rightarrow x=\begin{cases}4.73\\1.27\end{cases}$$

2. (50 points) Consider a binary classification problem with likelihoods $P(x|C_1)=x+\frac{1}{2}$, $P(x|C_2)=\frac{3x^2}{4}+\frac{3}{4}$ both for $x\in[0,1]$. Assume $P(C_2)=0.25$, and the cost matrix $\lambda=\begin{bmatrix}1 & 2\\ 3 & 1\end{bmatrix}$. Find the classifier which minimizes the conditional risk.

Sol:

$$\frac{p(x|C_1)}{p(x|C_2)}\overset{w_1}{>}\frac{p(C_2)}{p(C_1)}\frac{\lambda_{12}-\lambda_{22}}{\lambda_{21}-\lambda_{11}}\Rightarrow\frac{x+1/2}{\frac{3}{4}x^2+3/4}\overset{w_1}{>}\frac{1/4}{3/4}\frac{(2-1)}{(3-1)}$$

$$\Rightarrow x+1/2\overset{w_1}{>}\frac{1}{6}(\frac{3}{4}x^2+3/4)\Rightarrow x^2-8x-3\overset{w_1}{<}0\Rightarrow\frac{8-\sqrt{76}}{2}<x<\frac{8+\sqrt{76}}{2}$$

all samples will be classified as C_1 with respect to range of x.