In The Name of God, The Compassionate, The Merciful

Name: ....

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Statistical Pattern Recognition (CE-725)
Department of Computer Engineering
Quiz #6 (LDF and non-parametric modeling) - Spring 2012

1. (30 points) Given the following set of prototypes:

S1: (0,1), (0,2)

S2: (1,0), (2,0)

Apply pseudo-inverse procedure to find a solution vector for a linear discriminant function.

Hint : if $A = \begin{bmatrix} 5 & 0 & 3 \\ 0 & 5 & 3 \\ 3 & 3 & 4 \end{bmatrix}, then, A^{-1} = \begin{bmatrix} 1.1 & 0.9 & -1.5 \\ 0.9 & 1.1 & -1.5 \\ -1.5 & -1.5 & 2.5 \end{bmatrix}$

Sol:

The pseudo-inverse weights vector obtaining from $w = (X^T X)^{-1} X^T b$, then :

$$X = \begin{bmatrix} 0\,1\,1 \\ 0\,2\,1 \\ 1\,0\,1 \\ 2\,0\,1 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \Rightarrow X^T X = \begin{bmatrix} 5 & 0 & 3 \\ 0 & 5 & 3 \\ 3 & 3 & 4 \end{bmatrix}, X^T b = \begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix}$$

$$\Rightarrow w = (X^T X)^{-1} X^T b = \begin{bmatrix} -0.6 \\ 0.6 \\ 0 \end{bmatrix}$$

Hence, the discriminant boundary will be as X1=X2

2. (30 points) What is the most critical parameter in the Parzen window approach to density estimation? What procedure would you adopt to select this parameter?

Sol:

The window width, h_n, is the most critical parameter in the Parzen window approach. This parameter can be selected by cross-validation where a portion of the training set Is used to form a validation set. The classifier is trained on the remaining patterns in the training set for different values of h_n. The h_n that results in the smallest error in the Validation set is selected as the most optimal one.

3. (40 points) Consider a set of one-dimensional values sampled from an unknown density p(x): 1, 1.5,1.75, 2, 2.5, 2.75, 3, 5, 6, 6.25, 6.5, 7, 7.5. Estimate the value of the density function, $\hat{P}(x)$, At x=0, 1, 3, 5, 7 and 9, using a Parzen window with window width 1.

Sol:

$$\hat{P}(x) = \frac{k_n / n}{v_n}$$

Here, n=13, h_n=1 and v_n=1,k_n can be computed by using the expression

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$$k_n = \sum_{i=1}^{n} \phi(\frac{x - x_i}{h_n}),\ \phi(u) = \begin{cases} 1, & if\ |u| \le 1/2 \\ 0, & otherwise. \end{cases}$$

Thus,

$$\hat{P}(0) = 0, \hat{P}(1) = 2/13, \hat{P}(3) = 3/13, \hat{P}(5) = 1/13, \hat{P}(7) = 3/13, \hat{P}(9) = 0.$$