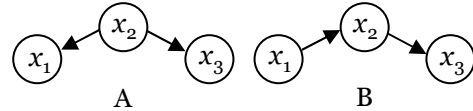


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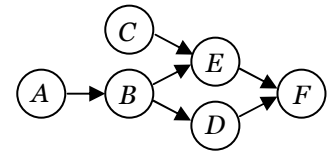
Statistical Pattern Recognition (CE-725)
Department of Computer Engineering
Mini Exam #2 (Graphical Models) - Spring 2012
100 points – 50 minutes

1. (10 points) Consider the following Bayesian nets, which all their variables are binary. Having $P_A(x_1=1, x_2=1, x_3=1) = 0.1$, find $P_B(x_1=1, x_2=1, x_3=1)$.



2. (30 points) This problem will concern the following Bayesian net. All variables in this network are Boolean, and,

- Nodes with a single parent take the value of their parent with probability 3/4 otherwise they take the other value.
- Nodes with two parents take the value of the first parent with probability 1/2 otherwise they take the value of the second parent.
- $P(A=1)=p, P(C=1)=q$.



- a. If some node X has a single parent Y , and $P(Y=1) = a$, what is a simple expression for $P(X=1)$? Assume that there is no child of X to worry about.
- b. If some node X has a two independent parents Y, Z , and $P(Y=1)=a, P(Z=1)=b$, what is a simple expression for $P(X=1)$? Assume that there is no child of X to worry about.
- c. What is $P(F=1)$ in the above graph?

3. (20 points) Consider an HMM with states $\{w_0, w_1, w_2\}$, observations $\{v_0, v_1, v_2\}$, transition probabilities a_{ij} and symbol probabilities b_{jk} .

$$a_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0.2 & 0.3 & 0.5 \\ 0.4 & 0.5 & 0.1 \end{pmatrix}$$

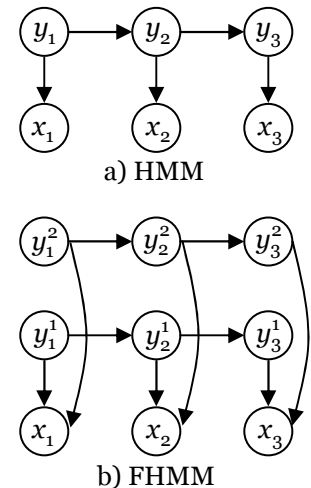
$$b_{jk} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.7 & 0.3 \\ 0 & 0.4 & 0.6 \end{pmatrix}$$

- a. Suppose the initial hidden state at $t=0$ is w_1 . Starting from $t=1$, what is the probability of generating the particular sequence $\{v_2, v_1, v_0\}$ by the given HMM?
- b. Given the above sequence $(\{v_2, v_1, v_0\})$, what is the most probable sequence of hidden states?

4. (40 points) Consider the hidden Markov model in Fig. (a) that consists of three observations $\{x_1, x_2, x_3\}$ and three hidden states $\{y_1, y_2, y_3\}$.

a. Write down the joint probability distribution $P(x_1, x_2, x_3, y_1, y_2, y_3)$.

b. In an HMM, the observations are generated from a sequence of hidden states. However, in some applications, like speech, biology and NLP, the observations are known to be generated from the interaction of multiple independent state sequences. This gives rise to the Factorial HMM (FHMM) model depicted in Fig. (b). Fig. (b) shows two independent state sequences: $\{y_1^1, y_2^1, y_3^1\}$ and $\{y_1^2, y_2^2, y_3^2\}$. At each time step, say t , the observation, x_t is generated based on the values of the hidden states in both sequences: y_t^1 and y_t^2 . Write down the joint probability $P(x_1, x_2, x_3, y_1^1, y_2^1, y_3^1, y_1^2, y_2^2, y_3^2)$.



c. The FHMM is an example of a Bayesian network. Use the conditional independence property in HMM or Bayesian networks to give a simpler expression of the conditional probability $P(y_3^1, y_1^2 | y_2^1, y_2^2)$.

d. Can we write $P(y_2^1, y_2^2 | x_1, x_2, x_3)$ as $P(y_2^1 | x_1, x_2, x_3)P(y_2^2 | x_1, x_2, x_3)$? Explain your answer.

e. Can we write $P(y_2^1, y_2^2)$ as $P(y_2^1)P(y_2^2)$? Explain your answer.