3.26

(a) If $x(t)$ is real, then $x(t) = x^*(t)$. This implies that for $x(t)$ real $a_k = a^*_{-k}$. Since this is not true in this case problem, $x(t)$ is not real.

(b) If $x(t)$ is even, then $x(t) = x(-t)$ and $a_k = a_{-k}$. Since this is true for this case, $x(t)$ is even.

(c) We have

$$g(t) = \frac{dx(t)}{dt} \stackrel{FS}{\longleftrightarrow} b_k = jk\frac{2\pi}{T_0}a_k.$$

Therefore,

$$b_k = \begin{cases} 0, & k = 0 \\ -k(1/2)^{|k|}(2\pi/T_0), & \text{otherwise} \end{cases}.$$

Since b_k is not even, $g(t)$ is not even.

3.37

The frequency response of the system may be easily shown to be

$$H(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} - \frac{1}{1-2e^{-j\omega}}.$$

(a) The Fourier series coefficients of $x[n]$ are

$$a_k = \frac{1}{4}, \qquad \text{for all } k.$$

Also, $N = 4$. Therefore, the Fourier series coefficients of $y[n]$ are

$$b_k = a_k H(e^{j2k\pi/N}) = \frac{1}{4}\left[\frac{1}{1-\frac{1}{2}e^{-j\pi k/2}} - \frac{1}{1-2e^{-j\pi k/2}}\right].$$

(b) In this case, the Fourier series coefficients of $x[n]$ are

$$a_k = \frac{1}{6}[1 + 2\cos(k\pi/3)], \qquad \text{for all} k.$$

Also, $N = 6$. Therefore, the Fourier series coefficients of $y[n]$ are

$$b_k = a_k H(e^{j2k\pi/N}) = \frac{1}{6}[1 + 2\cos(k\pi/3)]\left[\frac{1}{1-\frac{1}{2}e^{-j\pi k/3}} - \frac{1}{1-2e^{-j\pi k/3}}\right].$$

3.58

(a) We have

$$z[n+N] = \sum_{<L>} x[r]y[n+N-r].$$

Since $y[n]$ is periodic with period N, $y[n+N-r] = y[n-r]$. Therefore,

$$z[n+N] = \sum_{<L>} x[r]y[n-r] = z[n].$$

Therefore, $z[n]$ is also periodic with period N.

(b) The FS coefficients of $z[n]$ are

$$\begin{aligned} c_l &= \frac{1}{N}\sum_{n=<N>}\sum_{k=<N>} a_k b_{n-k} e^{-j2\pi nl/N} \\ &= \frac{1}{N}\sum_{k=<N>} a_k e^{-j2\pi kl/N} \sum_{n=<N>} b_{n-k} e^{-j2\pi(n-k)l/N} \\ &= \frac{1}{N} N a_l N b_l \\ &= N a_l b_l. \end{aligned}$$

(c) Here, $n = 8$. The nonzero FS coefficients in the range $0 \le k \le 7$ for $x[n]$ are $a_3 = a_5^* = 1/2j$. Note that for $y[n]$, we need only evaluate b_3 and b_5. We have

$$b_3 = b_5^* = \frac{1}{4(1 - e^{-j3\pi/4})}.$$

Therefore, the only nonzero FS coefficients in the range $0 \le k \le 7$ for the periodic convolution of these signals are $c_3 = 8a_3b_3$ and $c_5 = 8a_5b_5$.

(d) Here,

$$x[n] \overset{FS}{\longleftrightarrow} a_k = \frac{1}{16j}\left[\frac{1 - e^{j(3\pi/7 - \pi k/4)4}}{1 - e^{-j(3\pi/7 - \pi k/4)}} - \frac{1 - e^{j(3\pi/7 + \pi k/4)4}}{1 - e^{-j(3\pi/7 + \pi k/4)}}\right]$$

and

$$y[n] \overset{FS}{\longleftrightarrow} b_k = \frac{1}{8}\left[\frac{1 - (1/2)^8}{1 - (1/2)e^{-jk\pi/4}}\right].$$

Therefore,

$$z[n] = x[n]y[n] \overset{FS}{\longleftrightarrow} 8a_k b_k.$$

3.62

(a) The fundamental period of the input is $T = 2\pi$. The fundamental period of the input is $T = \pi$. The signals are as shown in Figure S3.62.

(b) The Fourier series coeffients of the output are

$$b_k = \frac{2(-1)^k}{\pi(1-4k^2)}.$$

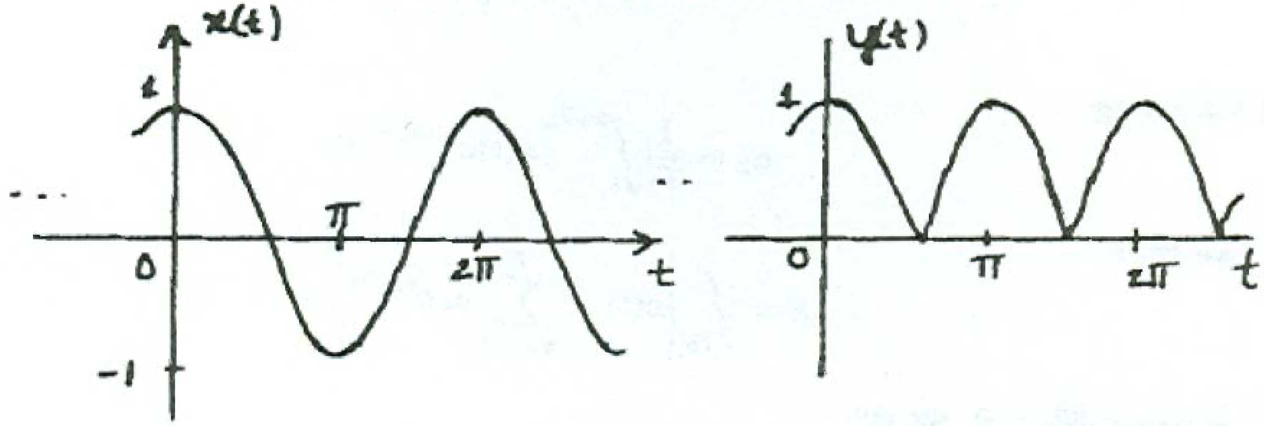


Figure S3.62

(c) The dc component of the input is 0. The dc component of the output is $2/\pi$.

problem1 : repetitive!

: problem 2

می‌دانیم که ضرایب سری فوریه تابع پالس متقارن متناوب به صورت $\frac{2AT_1}{T_0}\,\text{Sinc}\,\frac{2kT_1}{T_0}$ می‌باشد. همچنین $\delta[k]$ ضریب فوریه‌ی سیگنال ثابت است که فقط a_0 آن مقدار دارد. $1 \xleftrightarrow{\mathcal{I}} a_k=\delta[k]$

①

$$x_1(t): \frac{2}{7}\,\text{Sin}\,\frac{2k}{7} \Rightarrow \frac{T_1}{T_0}=\frac{1}{7}\ ,\ A=1 \Rightarrow T_1=2,\ T_0=14$$

②

$$x_2(t):\quad \frac{2}{14}\,\text{Sinc}\,\frac{k}{7} \Rightarrow \frac{T_1}{T_0}=\frac{1}{14} \Rightarrow T_1=1,\ T_0=14\ ,\ A=1$$

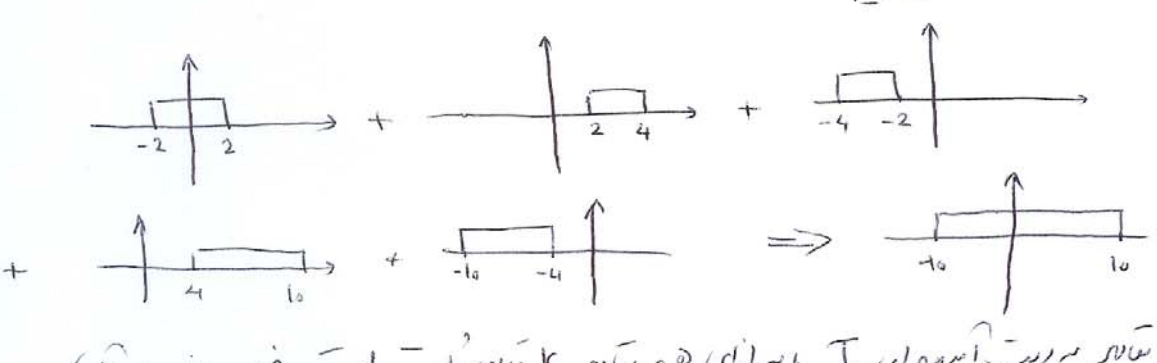
$$2\cos\frac{3\pi k}{7} = e^{\frac{3jk\pi}{7}} + e^{\frac{-3jk\pi}{7}} \Rightarrow$$ شیفت یافته‌ی پالس به اندازه‌ی ۳ و ۳-

③

$$x_3(t):\quad \frac{3}{14}\,\text{Sinc}\,\frac{3k}{7} \Rightarrow T_1=3,\ T_0=14\ ,\ A=1/2$$

$$2\cos k\pi = e^{jk\pi} + e^{-jk\pi} \Rightarrow$$ شیفت یافته‌ی پالس به اندازه‌ی 7 و 7-

شکل توابع x_1 و x_2 و x_3 را در زیر می‌بینیم:



که با توجه به مقادیر به دست آمده برای T_0 و به ازای همه‌ی مقادیر k تابعی ثابت است و ضریب فوریه‌ی آن $\delta[k]$ می‌باشد.

: problem 3

a) $$H(j\omega) = \frac{1+2j\omega}{(j\omega)^2+4j\omega+3} \Rightarrow h(t) = \frac{1}{4}e^{-t} + \frac{3}{2}e^{-3t}$$

b) $$x(t) * h(t) \xleftrightarrow{\mathcal{I}} X(j\omega)H(j\omega)$$

$$X(j\omega) = \frac{2}{2j\omega+1} \Rightarrow Y(j\omega) = \frac{2}{1+j\omega} + \frac{2}{3+j\omega} \Rightarrow y(t) = 2(e^{-t}+e^{-3t}$$

c) $x(t) = 2 \implies X(j\omega) = 2\,\delta(k)$

راه اول - با استفاده از فوریه : $Y(j\omega) = 2\,\delta(j\omega)\left(\frac{1/2}{3+j\omega} + \frac{3/2}{1+j\omega}\right) = \frac{20}{6}\,\delta(k)$

$\implies y(t) = 20/6$

راه دوم - کانولوشن : $y(t) = h(t) * x(t) = 2\int_0^{\infty} e^{-3t} + e^{-t} = 10/3$

d) $Y(j\omega) = \frac{4}{(1+j\omega)(3+j\omega)} \implies |Y(j\omega)|^2 = \frac{16}{(1+\omega^2)(9+\omega^2)} = \frac{16}{9+10\omega^2+\omega^4}$

$2\pi\left(4\int_0^{\infty} e^{-2t}\,dt + 4\int_0^{\infty} e^{-6t}\,dt\right) = 4\pi + \frac{4}{3}\pi = \frac{16\pi}{3}$

:Problem4

Since $a_k = a_{-k}$, we require that $x(t) = x(-t)$. Also, note that since $a_k = a_{k+2}$, we require that

$$x(t) = x(t)e^{-j(4\pi/3)t}.$$

This in turn implies that $x(t)$ may have nonzero values only for $t = 0, \pm 1.5, \pm 3, \pm 4.5, \ldots$ Since $\int_{-0.5}^{0.5} x(t) = 1$, we may conclude that $x(t) = \delta(t)$ for $-0.5 \le t \le 0.5$. Also, since $\int_{0.5}^{1.5} x(t)dt = 2$, we may conclude that $x(t) = 2\delta(t-3/2)$ in the range $0.5 \le t \le 3/2$. Therefore, $x(t)$ may be written as

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-k3) + 2\sum_{k=-\infty}^{\infty} \delta(t-3k-3/2).$$