Signals and Systems September 25, 2011
Sharif University of Technology CE 40-242
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Date Due: Mehr 18, 1390

Homework 2 (Chapters 1 & 2)

Problems

1. Determine and sketch the following signal and its even and odd parts. Label your sketches carefully.
   $x(t) = u(t+1) - u(t) + (u(t) - u(t-1)) * (u(t-1) - u(t-2)) * \delta(t-1)$

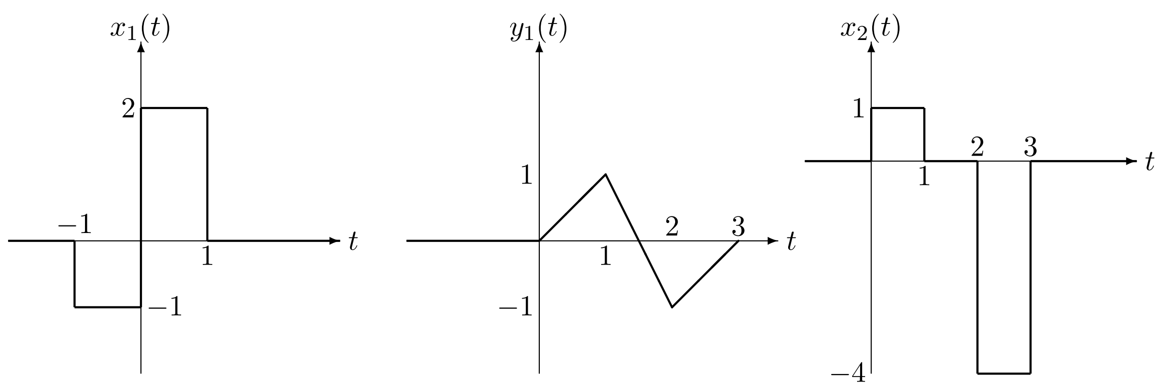
2. Are the following signals periodic? If so determine their fundamental period.

   a. $w(t) = \cos(\frac{\pi}{6}t) + \cos(\frac{3\pi}{5}t)$
   b. $z(t) = \sin(\frac{\pi}{6}t) + \cos(t)$

3. A system may or may not be linear, time-invariant, memoryless, causal, or stable. Determine whether or not each of the following systems has these properties.

   a. $y(t) = \sum_{i=-5}^{5} a_i x(t-i)$
   b. $y[n] = \begin{cases} (-1)^n x[n] & \text{if } x[n] \geq 0 \\ 2x[n] & \text{if } x[n] < 0 \end{cases}$
   c. $y[n] = \mathcal{O}dd\{x[n-1]\}$

4. Consider an LTI system whose response to the signal $x_1(t)$ is the signal $y_1(t)$ where these signals are depicted below. Determine and provide a labeled sketch of the response to the input $x_2(t)$, which is also depicted below.



5. Compute the convolution sum $y[n] = x[n] * h[n]$ for each of the following pairs of signals.

   a. $x[n] = h[n] = \alpha^n u[n]$
   b. $x[n] = (-\frac{1}{4})^n u[n-4]$, $h[n] = 2^n u[3-n]$

6. Compute the convolution $y(t) = x(t) * h(t)$ for each of the following pairs of signals.

   a. $x(t) = h(t) = e^{-\alpha t} u(t)$

b. $x(t) = x_1(t)$ (from Prob. 6), $h(t) = (u(t) - u(t-1)) * (u(t) - u(t-1))$

7. The following are the impulse responses of LTI systems. Determine whether each system is causal and/or stable. Justify your answers. (P 2.28 e,g and 2.29 e,f)

   a. $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[n-1]$

   b. $h[n] = n(\frac{1}{3})^n u[n-1]$

   c. $h(t) = e^{-6|t|}$

   d. $h(t) = te^{-t}u(t)$

8. P 2.40 p. 148

9. P 2.47 p. 152

10. P 2.64 a,b,d p. 166

11. Let $\mathbf{V}$ be a subspace of Hilbert space $\mathbf{H}$. A projector $P_{\mathbf{V}}$ on $\mathbf{V}$ is a linear operator that satisfies:

    1. $\forall f \in \mathbf{H} \rightarrow P_{\mathbf{V}} f \in \mathbf{V}$

    2. $\forall f \in \mathbf{V} \rightarrow P_{\mathbf{V}} f = f$

    The projector $P_{\mathbf{V}}$ is orthogonal if $\forall f \in \mathbf{H}, \forall g \in \mathbf{V} \rightarrow \langle f - P_{\mathbf{V}} f, g \rangle = 0$. If $P_{\mathbf{V}}$ is an orthogonal projector on $\mathbf{V}$ prove that:

    a. $\forall f \in \mathbf{H} \rightarrow ||f - P_{\mathbf{V}} f|| = \min_{g \in \mathbf{V}} ||f - g||$.

    b. if $\{e_n\}_n \in \mathcal{N}$ is an orthogonal basis of $\mathbf{V}$, then $P_{\mathbf{V}} f = \sum_{n=0}^{+\infty} \frac{\langle f, e_n \rangle}{||e_n||^2} e_n$

Practical Assignment

1. Define the MATLAB vector `nx` to be the time indices $-3 \leq n \leq 7$ and the MATLAB vector `x` to be the values of the signal $x[n]$ at those samples, where $x[n]$ is given by

$$x[n] = \begin{cases} 2, & n = 0, \\ 1, & n = 2, \\ -1, & n = 3, \\ 3, & n = 4, \\ 0, & \text{otherwise} \end{cases}$$

   If you define these vectors correctly you should be able to plot this discrete-time sequence using `stem(nx,x)`. Now plot $x[n-2]$ and $x[-n+1]$.

2. Consider the impulse response $h[n] = \delta[n+1] + \delta[n-1]$ and the input $x[n] = \delta[n] - 3\delta[n-2]$. In MATLAB define the vectors `h` and `x` corresponding to these sequences. Use `conv` (read help for `conv`: `help conv`) to compute the output signal $y[n]$. Determine a vector of time indices corresponding to y and store those in the vector `ny`. Plot $y[n]$ as a function of n using the command `stem(ny,y)`.

3. Consider the impulse response $h[n]$ and input $x[n]$ defined below

$$h[n] = u[n+2]$$
$$x[n] = (\tfrac{3}{5})^{n-2} u[n-2]$$

   a. Analytically compute and sketch the convolution of $h[n]$ and $x[n]$.

b. Define a vector `h` in MATLAB that contains the values of $h[n]$ for $2 \leq n \leq 14$, and define a vector `x` in MATLAB that contains the values of $x[n]$ for $0 \leq n \leq 24$. Define vectors `nh` and `nx` containing the corresponding time indices. Compute the convolution of these two finite-length vectors using `y=conv(x,h)`. Compute the vector of time indices for `y` and store that in `ny`. Plot `y` using the `stem` command. Note that since the `h` and `x` vectors are shortened versions of the true signals, only a portion of the output vector will contain the true values of $y[n]$. *Specify which values in the output vector* `y` *are correct and which are not. Explain your answer.*