

Date Due: Aban 7, 1390

## Homework 3 (Chapter 3)

### Problems

1. Determine the fundamental frequency, fundamental period, and Fourier series coefficients for the following signals:
  - a. Signals depicted in parts b,d, and e of Figure P3.22 of textbook p. 256
  - b. Signals depicted in parts a and b of Figure P3.28 of textbook p. 258
2. Let  $x(t)$  and  $y(t)$  be periodic signals whose fourier series coefficients are  $a_k$  and  $b_k$  respectively

$$a_k = \begin{cases} 2, & k = 0 \\ (\frac{1}{2})^{|k|}, & \text{otherwise} \end{cases}, b_k = \begin{cases} \frac{1}{2}, & k = 0 \\ \frac{(-j)^k \sin(k\pi/2)}{k\pi}, & \text{otherwise} \end{cases}$$

Use Fourier series properties to answer the following questions, then find and sketch  $y(t)$  ( $T = 4$ ).

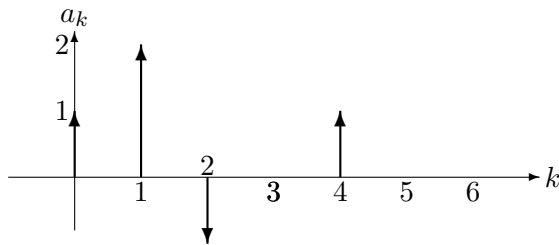
- a. Is  $x(t)$  real?
  - b. Is  $x(t)$  even?
  - c. Is  $dx(t)/dt$  even?
3. Let  $x(t)$  and  $y(t)$  be periodic signals with fundamental period  $T$  whose fourier series coefficients are  $a_k$  and  $b_k$  respectively.

a. Show that

$$\frac{1}{T} \int_T x(t)y^*(t) = \sum_{k=-\infty}^{\infty} a_k b_k^* \tag{1}$$

b. If  $T_x, T_y \in \mathbf{Q}$  are the fundamental periods of  $x(t)$  and  $y(t)$  respectively. Derive an equality similar to part a.

4. Let  $x(t)$  be a periodic signal with period  $T = 5$  and Fourier series coefficients  $a_k$  depicted as below.

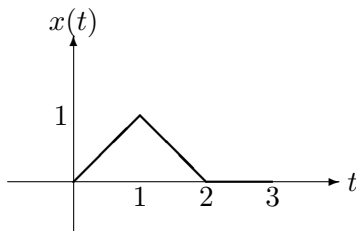


- a. Find  $x(0)$  without calculating  $x(t)$ .

b. Evaluate the following sums without calculating  $x(t)$ .

$$\int_T x(t)e^{-j\frac{4\pi}{5}t}dt, \int_T x^2(t-2)dt, \int_T x^2(t)e^{-j\frac{4\pi}{5}t}dt$$

5.  $h(t) = e^{-2t}u(t)$  and  $x(t)$  is sketched below.



a. Compute  $y(t) = x(t) * h(t)$ .

b. Compute  $g(t) = \frac{dx(t)}{dt} * h(t)$ .

c. What is the relationship between  $g(t)$  and  $y(t)$ ?

6. Consider a continuous-time LTI system  $S$  whose frequency response is

$$H(j\omega) = \begin{cases} 1, & |\omega| \leq 140 \\ 0, & \text{otherwise} \end{cases}$$

When the input to the system is a signal  $x(t)$  with fundamental period  $T = \pi/7$  and fourier series coefficients  $a_k$ , it is found that the output  $y(t)$  is identical to  $x(t)$ . For what values of  $k$  is it guaranteed that  $a_k = 0$ ? (P 3.35 p. 260)

7. Consider a discrete-time LTI system with impulse response

$$h[n] = \begin{cases} 1, & 0 \leq n \leq 2 \\ -1, & -2 \leq n \leq -1 \\ 0, & \text{otherwise} \end{cases}$$

Given that the input to this system is

$$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n - 4k],$$

determine the Fourier series coefficients of the output  $y[n]$ . (P 3.38 p. 261)

8. Let  $x[n]$  be a periodic sequence with period  $N$  and Fourier series representation

$$x[n] = \sum_{\langle N \rangle} a_k e^{jk(2\pi/N)n},$$

The Fourier series coefficients of each of the following signals can be expressed in terms of  $a_k$ . Derive the expressions. (P 3.48 p. 256)

a.  $x[n - n_0]$

b.  $x[n] - x[n - 1]$

c.  $x[n] - x[n - \frac{N}{2}]$  (assume that  $N$  is even)

- d.  $x[n] - x[n + \frac{N}{2}]$  (assume that  $N$  is even; note that this signal is periodic with period  $N/2$ )
- e.  $x^*[-n]$
- f.  $(-1)^n x[n]$  (assume that  $N$  is even)
- g.  $(-1)^n x[n]$  (assume that  $N$  is odd; note that this signal is periodic with period  $2N$ )
- h.  $y[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

### Practical Assignment

1. Consider the periodic square wave  $x(t) = \begin{cases} t + \frac{3}{2}, & -1 \leq t \leq 0 \\ -t + \frac{3}{2}, & 0 \leq t \leq 1 \\ 0 & 1 \leq t \leq 3 \end{cases}$  defined over one period. Obtain a general expression for its Fourier series coefficients. Calculate the first 100 coefficients in MATLAB. Display the reconstructed signal using the first 1, 3, 7, 19, 41, and 79 coefficient(s) overlaid on the original signal. Calculate the Mean Square Errors (MSEs) between these reconstructed signals in all 100 possible partial summations and the original signal. Plot the MSE versus number of coefficients.
2. Consider the periodic signal depicted in Figure P3.22 (c) on p. 255 of your textbook. Obtain a general expression for its Fourier series coefficients. Calculate the first 100 coefficients in MATLAB. Display the reconstructed signal using the first 3, 5, and 10 coefficients overlaid on the original signal. Overlay each reconstruction on the original signal with different color or style. How many coefficients are needed for the reconstructed signal to be an appropriate approximation of the original signal by inspection? Display this reconstruction as well. Calculate the MSEs between these reconstructed signals in all 100 possible partial summations and the original signal. Plot the MSE versus number of coefficients.

(The MSE is defined as  $\frac{\sum_{k=1}^N (y[k] - x[k])^2}{N}$ )