Signals and Systems October 22, 2011
Sharif University of Technology CE 40-242
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Date Due: 14th Aban 1390

Homework 4 (Chapter 4)

Problems

1. Compute the Fourier transform of each of the following signals: (P 4.21 (a, c, f, g, i) p. 338 and 2 extra parts.)

   a. $x(t) = e^{-|t|} \cos 2t$

   b. $[e^{-\alpha t} \cos \omega_0 t] u(t), \alpha > 0$

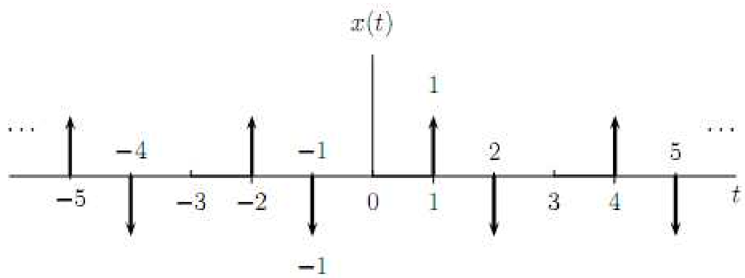
   c. $x(t) = (1 - |t|)u(t + 1)u(1 - t)$

   d. $x(t) = [\frac{\sin \pi t}{\pi(t-1)}][\frac{\sin 2\pi(t-1)}{\pi t}]$

   e. $x(t)$ as shown in Figure P4.21(a) in p. 338 of textbook.

   f. $x(t) = \begin{cases} 1 + t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

   g. The signal $x(t)$ depicted below:



2. Determine the continuous-time signal corresponding to each of the following transforms:

   a. $X(j\omega) = 5[\delta(\omega + 1) - \delta(\omega - 1)] - 2j[\delta(\omega - \pi) + \delta(\omega + \pi)]$

   b. $X(j\omega) = 2\cos(3\omega - \pi/3)$

3. Determine which, if any, of the real signals depicted in below have Fourier transforms that satisfy each of the following conditions:

   1. $\mathcal{R}e\{X(j\omega)\} = 0$

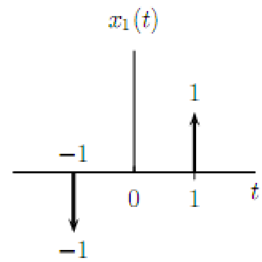
   2. $\mathcal{I}m\{X(j\omega)\} = 0$

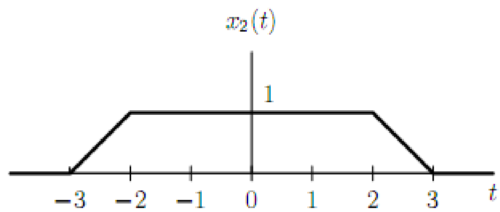
   3. There exists a real α such that $e^{j\alpha\omega}X(j\omega)$ is real

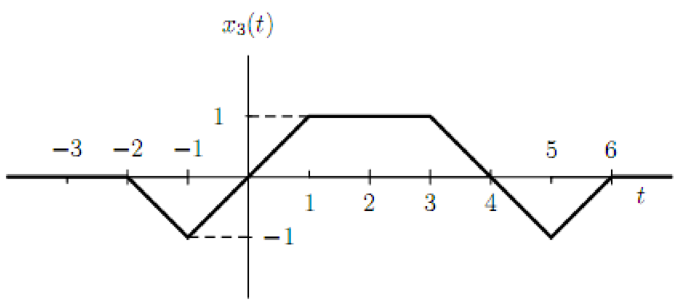
   4. $\int_{-\infty}^{\infty} X(j\omega)d\omega = 0$

   5. $\int_{-\infty}^{\infty} \omega X(j\omega)d\omega = 0$

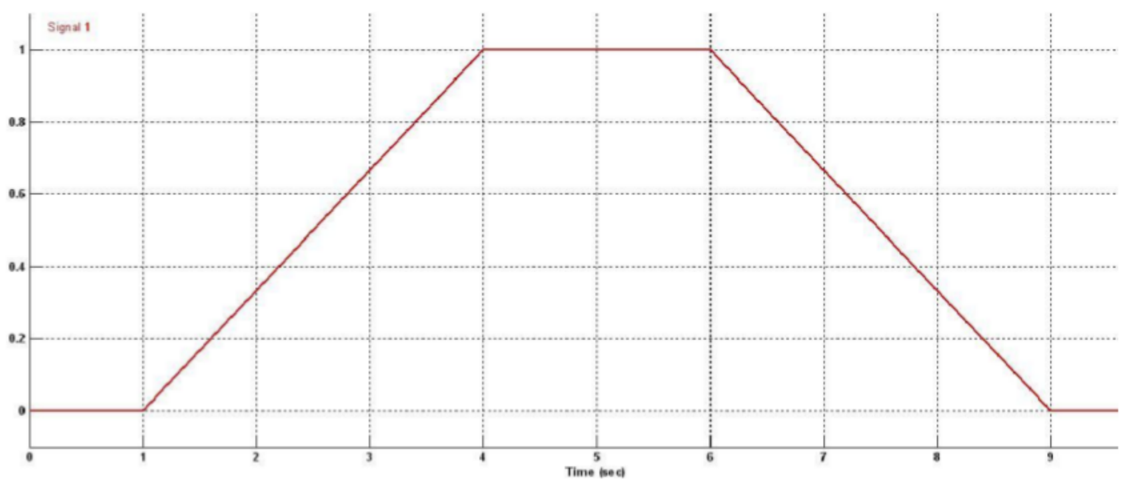
   6. $X(j\omega)$ is periodic







4. Let $X(j\omega)$ denote the Fourier Transform of the signal $x(t)$ depicted below.



a. Find $\sphericalangle X(j\omega)$.

b. Find $X(0)$.

c. Find $\int_{-\infty}^{\infty} X(j\omega) d\omega$.

d. Evaluate $\int_{-\infty}^{\infty} X(j\omega) \frac{2 \sin \omega}{\omega} e^{j2\omega} d\omega$.

e. Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$.

f. Sketch the inverse Fourier transform of $\mathcal{R}e\{X(j\omega)\}$.

Note: You should perform all these calculations without explicitly evaluating $X(j\omega)$.

5. Find the impulse response of a system with the following frequency response: (P 4.18 p. 337)

$$H(j\omega) = \frac{(\sin^2(3\omega))\cos\omega}{\omega^2}.$$

6. Consider a causal, discrete-time LTI system $F : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$ whose input and output signals x and y, respectively, satisfy the following linear, constant-coefficient difference equation:

$$y(n) + \alpha^2 y(n-2) = x(n) + x(n-2)$$

where $0 < \alpha < 1$.

a. Determine an expression for the frequency response $F : \mathbb{R} \to \mathbb{C}$ of the system.

b. Suppose $\alpha = 0.95$. Using a geometric (graphical) analysis, provide a well-labeled sketch of $|F(j\omega)|$, the magnitude of the frequency response of the system. Explain why this filter is called a *notch* filter.

c. Suppose $\alpha = 0.95$ and that the input signal is characterized by

$$\forall n \in \mathbb{Z}, x(n) = \cos(\frac{\pi}{2}n) + \frac{1}{3}\sin(\frac{\pi}{4}n) + 3 + (-1)^n.$$

Using little to no mathematical manipulation, determine a reasonable approximation for the corresponding output signal values $y(n)$. What assumption did you have to make about the phase response $\sphericalangle F$ that enabled you to approximate the output signal y? Why was your assumption about the phase reasonable?

7. **(Orthogonality-Preserving Property of the CTFT)** In this problem, we set out to prove that the continuous-time Fourier transform (CTFT) preserves mutual orthogonality of signals, and that the inverse of the CTFT preserves mutual orthogonality of signal spectra.
Consider a set $\phi_k, k \in \mathbb{Z}$, of mutually-orthogonal functions

$$\phi_k : \mathbb{R} \to \mathbb{C}$$

each of whose elements ϕ_k has finite energy E_ϕ, i.e.

$$\langle \phi_k, \phi_l \rangle \triangleq \int_{-\infty}^{\infty} \phi_k(t)\phi_l^*(t)dt = E_\phi \delta(k-l).$$

where δ is the Kronecker delta function and * denotes complex conjugation. Let $\hat{\phi}_k$ be the CTFT (spectrum) of ϕ_k, i.e., for $k \in \mathbb{Z}$,

$$\hat{\phi}_k : \mathbb{R} \to \mathbb{C}$$

$$\forall \omega \in \mathbb{R}, \hat{\phi}_k(\omega) = \int_{-\infty}^{\infty} \phi_k(t)e^{-j\omega t}dt$$

Show that

$$\langle \hat{\phi}_k, \hat{\phi}_l \rangle \triangleq \int_{-\infty}^{\infty} \hat{\phi}_k(\omega)\hat{\phi}_l^*(\omega)d\omega = 2\pi E_\phi \delta(k-l).$$

Practical Assignment

1. Consider a discrete-time system H_1 with impulse response

$$h_1[n] = \delta[n] + \delta[n-1] - \delta[n-2] - \delta[n-3],$$

a discrete-time system H_2 with impulse response

$$h_2[n] = (\frac{1}{2})^n (u[n+3] - u[n-3]),$$

and a discrete-time signal

$$x[n] = (\frac{1}{4})^n (u[n] - u[n-6]).$$

The signals $h_1[n]$, $h_2[n]$, and $x[n]$ are all defined for $-8 \leq n \leq 8$.

a. Plot $h_1[n]$, $h_2[n]$, and $x[n]$ together using the *subplot* function.

b. Consider a system H formed from the series connection of H_1 and H_2, where $x[n]$ is input to H_1, the output $v[n]$ of H_1 is input to H_2, and the output of H_2 is $y[n]$. Use the *conv* function to find $v[n]$ and $y[n]$. Plot $v[n]$ and $y[n]$ using the *subplot* function.

c. Now assume that the order of the systems is reversed, so that $x[n]$ is input to H_2, the output $v[n]$ of H_2 is input to H_1, and $y[n]$ is the output of H_1. Plot $v[n]$ and $y[n]$. Briefly explain why $v[n]$ is different in parts (b) and (c), whereas $y[n]$ is the same in both parts.