

Date Due: Azar 19, 1390

## Homework 6 (Chapter 7)

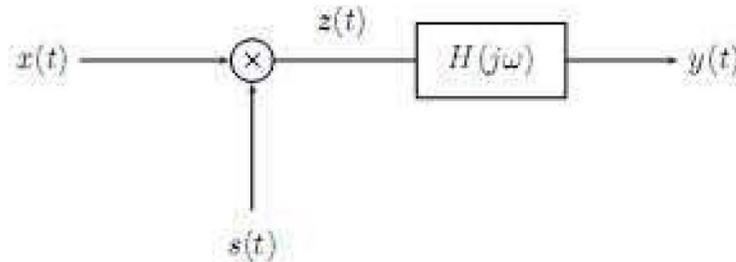
### Problems

- Suppose the impulse response of an ideal discrete-time lowpass filter with cutoff frequency  $\frac{\pi}{2}$  is interpolated (in accordance with Figure 7.37 on page 569 of textbook) to obtain an upsampling by a factor of 2. What is the frequency response corresponding to this upsampled impulse response? (P7.18 p. 560)
- Figure P7.28(a) (on page 566 of textbook) shows a system that converts a continuous-time signal to a discrete-time signal. The input  $x(t)$  is periodic with a period of 0.1 second. The Fourier series coefficients of  $x(t)$  are

$$a_k = \left(\frac{1}{2}\right)^{|k|}, -\infty < k < +\infty$$

The lowpass filter  $H(j\omega)$  has the frequency response shown in Figure P7.28(b) (on page 566 of textbook). The sampling period  $T = 5 \times 10^{-3}$  second. (P 7.28 p. 565)

- show that  $x[n]$  is a periodic sequence, and determine its period.
  - Determine the Fourier series coefficients of  $x[n]$ .
- A sinusoidal input signal,  $x(t) = \cos(10t)$  is sampled and filtered as shown below.

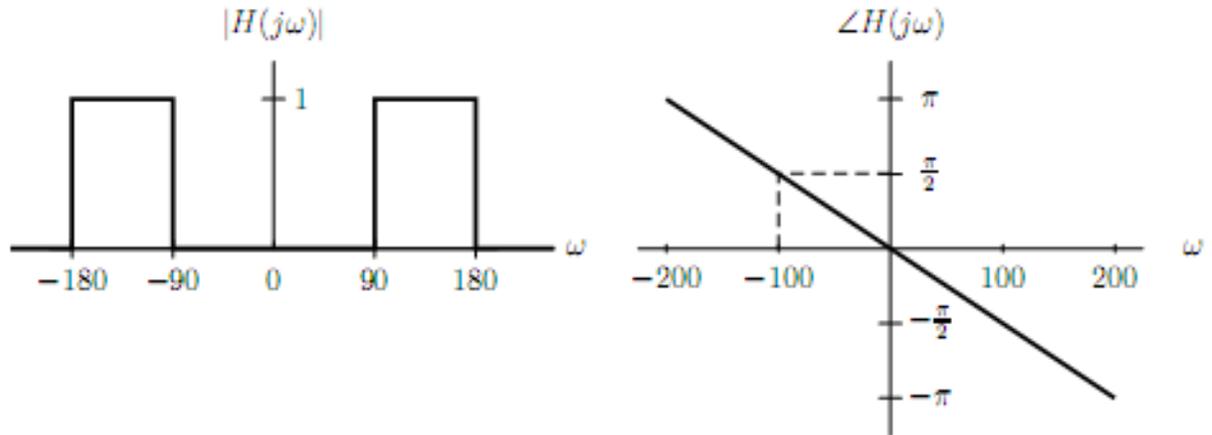


The frequency response of the filter is

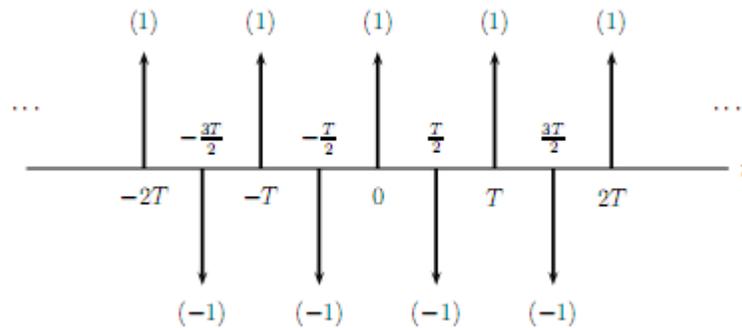
$$|H(j\omega)| = \begin{cases} 1, & 90 < |\omega| < 180 \\ 0, & \text{otherwise} \end{cases}$$

$$\angle H(j\omega) = -\frac{\pi\omega}{200}$$

as shown in the figure below:



- Suppose  $s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$  and  $T = \frac{2\pi}{90}$ . Provide a labeled sketch of  $Z(j\omega)$ , the Fourier transform of  $z(t)$ .
- Find  $y(t)$ , assuming the  $s(t)$  and the value of  $T$  given in part (a).
- The sampling function  $s(t)$  is changed to the form below, with  $T = \frac{2\pi}{90}$ .



Find  $y(t)$ .

- Figure P7.30 (on page 567 of textbook) shows a system consisting of a continuous-time LTI system followed by a sampler, conversion to a sequence, and an LTI discrete-time system. The continuous-time LTI system is causal and satisfies the linear, constant-coefficient differential equation

$$\frac{dy_c(t)}{dt} + y_c(t) = x_c(t).$$

The input  $x_c(t) = \delta(t - \frac{T}{2})$ . (P7.30 p. 567 with little change)

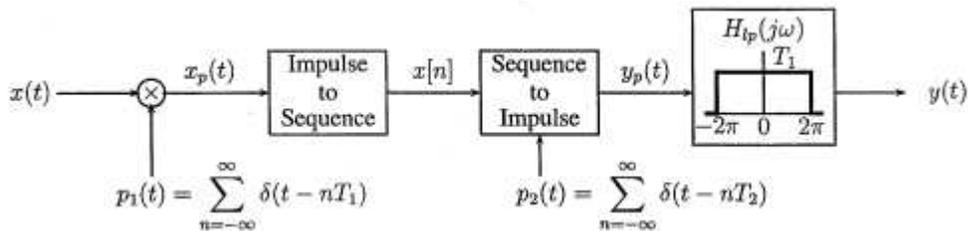
- Determine  $y_c(t)$ .
  - Determine the frequency response  $H(e^{j\omega})$  and the impulse response  $h[n]$  such that  $w[n] = \delta[n]$ .
- Shown in Figure P7.31 is a system that processes continuous-time signals using a digital filter  $h[n]$  that is linear and causal with difference equation

$$y[n] = \frac{3}{4}y[n-2] + x[n] + \frac{1}{4}x[n-1].$$

For input signals that are band limited such that  $X_c(j\omega) = 0$  for  $|\omega| > \frac{\pi}{T}$ , the system in the figure is equivalent to a continuous-time LTI system.

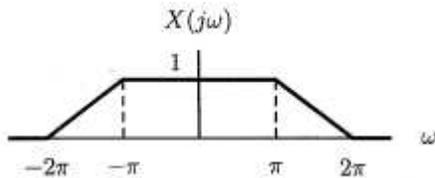
Determine the frequency response  $H_c(j\omega)$  of the equivalent overall system with input  $x_c(t)$  and output  $y_c(t)$ . (P7.31 p. 567 with little change)

6. Consider the following system:



$p_1(t)$  is an impulse train whose fundamental period is  $T_1$  and  $p_2(t)$  is another impulse train whose fundamental period is  $T_2$ .  $H_{lp}(j\omega)$  is a lowpass filter whose gain is  $T_1$  and cutoff frequency is at  $\omega_c$ . Note that  $x[n] = x(nT_1)$  and  $y_p(t) = \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT_2)$ .

The input  $x(t)$  is a band limited real signal whose Fourier transform is shown below:



a. Let's define

$$\begin{aligned} x_1[n] &= x(nT_1), \text{ where } T_1 = 1, \\ x_2[n] &= x(nT_1), \text{ where } T_1 = \frac{1}{3}. \end{aligned}$$

Provide the sketches of  $X_1(e^{j\omega})$  and  $X_2(e^{j\omega})$ , Fourier transforms of  $x_1[n]$  and  $x_2[n]$  respectively.

b. Suppose  $T_1 = \frac{1}{3}$  and  $T_2 = \frac{1}{2}$ . Provide a sketch of  $Y(j\omega)$ , Fourier transform of the overall output  $y(t)$ .

7. A signal  $x[n]$  with Fourier transform  $X(e^{j\omega})$  has the property that

$$\left( x[n] \sum_{k=-\infty}^{\infty} \delta[n - 3k] \right) * \left( \frac{\sin \frac{\pi}{3} n}{\frac{\pi}{3} n} \right) = x[n].$$

For what values of  $\omega$  is it guaranteed that  $X(e^{j\omega}) = 0$ ? (P7.33 p. 568)

### Practical Assignment

1. Start by writing a Matlab function file that produces samples of the sinusoidal signal,

$$x(t) = A \sin(2\pi f_0 t + \phi)$$

The samples are to be taken at periodic intervals of the sampling period,  $T_s = 1/f_s$ , where  $f_s$  is the sampling rate, and  $\phi$  can be any phase. The sampled signal is given by

$$x(nT_s) = A\sin(2\pi f_0 nT_s + \phi) = A\sin(2\pi \frac{f_0}{f_s} n + \phi).$$

Your function file should have inputs  $f_0$ ,  $A$ ,  $f_s$ ,  $\phi$ ,  $n_1$  and  $n_2$ , where  $n_1T_s$  is the starting time of the time interval of interest, and  $n_2T_s$  is the end time. The output of your function should be a vector of time samples, say *tvec*, and a vector, say *xnTs*, whose *i*th element is  $x((n_1 + i - 1)T_s)$ .

2. Set  $f_s = 8kHz$  and  $f_0 = 100Hz$ , and calculate  $x(nT_s)$  over the interval 0 to 10 ms. Plot  $x(nT_s)$  using the Matlab *stem* command. Make a choice for  $A$  and  $\phi$  and justify why these are reasonable choices in your report. Describe what you see. You can save an Encapsulated Postscript file using the command `print -deps mygraph.eps`.
3. When this sampled signal is transported to its destination, it is converted back into analog form and then transmitted out to the destination telephone. For simplicity we will assume that this digital-to-analog conversion is done by connecting the voice sample values with straight lines. This can be seen in Matlab simply by using the *plot* command rather than *stem*. In practice a more smooth reconstruction is done by performing analog filtering at the output. Over the interval from 0 to 10 ms, plot  $x(nT_s)$  using the *plot* command when  $f_0 = 100, 200, 400, 800Hz$ . Rather than plotting your graphs separately, plot all four output frequency plots on a single page using the *subplot* command. Label each sub-plot with its value of frequency,  $f_0$ .