Signals and Systems December 18, 2011
Sharif University of Technology CE 40-242
Dr. Hamid Reza Rabiee

Date Due: Exam date (Will *not* be postponed, so plan ahead)

Homework 8 (Chapter 10)

Problems

1. Determine the Z transform (including the region of convergence) for each of the following signals:
   (a) $x_1[n] = (\frac{1}{2})^n u[n-3]$
   (b) $x_2[n] = (1+n)(\frac{1}{3})^n u[n]$
   (c) $x_3[n] = n(\frac{1}{2})^{|n|}$
2. Determine and sketch all possible signals with Z transforms of the following forms. For each signal, indicate the associated region of convergence.
   (a) $X_1(z) = (\frac{1-z^2}{z})^2$
   (b) $X_2(z) = \frac{1-\frac{1}{3}z^{-1}}{1+\frac{1}{3}z^{-1}}$, $x(n)$ is right-sided.
   (c) $X_3(z) = \frac{1}{(1+z^{-1})^2(1-2z^{-1})(1-3z^{-1})}$, $1 < |z| < 2$.
   (d) $X_3(z) = \frac{z^3-2z}{z-2}$, $|z| < 2$.
3. Assume that a discrete-time LTI system has the input and output related by the following difference equation: $y[n] - 0.75[yn-1] + 0125y[n-2] = x[n]$.
   (a) Find y[n] by using Z transform when $x[n] = 1$ for $n = 0$ and $x[n] = 0$ for $n \neq 0$.
   (b) Verify the value of $y[0]$ in part (a) by using the initial-value theorem.
4. Consider a discrete-time LTI system with transfer function $H(z) = \frac{1-a^*z}{z-a}, |a| < 1$, where a^* represents the complex conjugate of a.
   (a) Sketch the pole-zero plot of $H(z)$ in the z-plane.
   (b) Is $H(z)$ stable and causal? Why?
   (c) Use the graphic method to show what the magnitude response of the system is.
5. A signal $\{y_k\}$ is related to another signal $\{x_k\}$ by means of the formula:
   $$y_k = \sum_{m=0}^{k} x_m, k \geq 0.$$
   If $Z\{y_k\} = \frac{1}{(z-1)^2}$ and $x_0 = 0$, what are the numerical values of x_1 and x_2?

6. **(Avoiding excitation)** Consider the system described by the following difference equation:

$$y[n] = x[n] + \frac{5}{2}y[n-1] - y[n-2]$$

Find an input $x[n]$ such that the output $y[n]$ is proportional to $(\frac{1}{n})^n$ for large values of n. Try to minimize the number of non-zero samples in $x[n]$.

7. Let $X(z)$ represent the Z transform of $x[n]$, and let $r_0 < |z| < r_1$ represent the region of convergence (ROC) of $X(z)$. Let $Y(z)$ represent the Z transform of $y[n] = 2^n(u[n] + x[n])$ where $u[n]$ represents the unit-step signal. Determine a closed-form expression for $Y(z)$ (which will depend on X) and find the region of convergence (ROC) for $Y(z)$.

8. Draw block diagram implementation of the following system as cascade of second-order sections with real-valued coefficients and depict the cascade form for this system.

$$H(z) = \frac{(1+3z^{-1})^2(1-\frac{1}{2}e^{j\pi/3}z^{-1})(1-\frac{1}{2}e^{-j\pi/3}z^{-1})}{(1-\frac{1}{2}e^{j\pi/5}z^{-1})(1-\frac{1}{2}e^{-j\pi/5}z^{-1})(1-\frac{5}{8}e^{j\pi/4}z^{-1})(1-\frac{5}{8}e^{-j\pi/4}z^{-1})}$$

9. (a) The input $x(n) = 2^n[u(n) - 3u(n-1)]$ to an unknown LTI system produces the output $y(n) = (3^n - 2^n)u(n)$.
   i. Determine the impulse response $h(n)$ of the system.
   ii. Is the solution unique? What if the system is known to be unstable? What if it is causal?

   (b) Determine the region of convergence of $Y(z)$ where
   i. $Y(z) = X_1(z) + X_2(z)$,

   $$X_1(z) = \frac{z}{z+1}, |z| > 1$$

   $$X_2(z) = \frac{z^{-2}}{z+1}, |z| > 1$$

   ii. $Y(z) = H(z)X(z)$,

   $$H(z) = \frac{1}{z^2+7z+10}, |z| > 5$$

   $$x(n) = \delta(n) + 2\delta(n-1)$$

   iii. $Y(z) = z^{-2}X(z)$,

   $$x(n) = 2^n u(-n-1)$$