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Time: 15 mins

Name:

Std. Number:

## Quiz 1 (Vector Spaces)

### Questions

1. define continuous-time, discrete-time and digital signal.
2. What is the difference between the following spaces?
  - Vector space
  - Banach space
  - Hilbert Space
3. Let  $B = \{b_1, b_2, \dots, b_m\}$  be a subset of  $\mathbb{R}^n$  which are pairwise orthogonal and none of them is zero. prove that they are linearly independent.

### Solutions

1. **continuous-time** A function with continuous domain and range  
**discrete-time** A function with discrete domain and continuous range  
**digital** A function with discrete domain and range
2. A vector space  $V$  is complete if every Cauchy sequence in  $V$ , converges to a vector in  $V$ . A complete vector space equipped with a norm is called a Banach space. A Banach space with an inner product is a Hilbert space.
3. Assume  $m$  variables  $\alpha_1, \alpha_2, \dots, \alpha_m$  exist (at least one of them is not zero) such that  $\alpha_1 b_1 + \alpha_2 b_2 + \dots + \alpha_m b_m = 0$  so  $\langle b_i, \alpha_1 b_1 + \alpha_2 b_2 + \dots + \alpha_m b_m \rangle = \alpha_i \|b_i\|^2 = 0 (\alpha_i \neq 0) \Rightarrow \|b_i\| = 0$  which is against hypothesis.