Signals and Systems 25. September 2011
Sharif University of Technology CE 40-242
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Time: 15 mins

Name: Std. Number:

Quiz 2 (Vector Space Bases) Solution

Questions

1. let $E = \{e_1, e_2, .., e_n\}$ be an orthonormal subset of Hilbert space H.

   a. Prove that for any vector $y \in H$,

   $$\|y\|_2^2 \geq \sum_{i=1}^{n} | < y, e_i > |^2$$

   let E_H be an orthonormal basis of H such that E is a subset of E_H. Then:

   $$\|y\|_2^2 = \sum_{e \in E_H} | < y, e > |^2 \geq \sum_{i=1}^{n} | < y, e_i > |^2$$

   b. Propose conditions on y so that equality holds?
   if $y \in Span(E)$ then equality holds.

2. If $E = \{e_1, e_2, .., e_n\}$ and $\tilde{E} = \{\tilde{e_1}, \tilde{e_2}, .., \tilde{e_n}\}$ are a pair of bioorthogonal bases of a Hilbert space H, prove that for any vector $y \in H$ the following equality holds.

   $$\|y\|_2^2 = \sum_i < y, e_i >< y, \tilde{e_i} >^*$$

   We know that

   $$y = \sum_i < y, e_i > \tilde{e_i}$$

   taking the inner product of both sides with y gives:

   $$\|y\|_2^2 = \sum_i < y, e_i >< \tilde{e_i}, y >= \sum_i < y, e_i >< y, \tilde{e_i} >^*$$