
Time: 15 mins

Name:

Std. Number:

Quiz 2 (Vector Space Bases)

Questions

1. let $E = \{e_1, e_2, \dots, e_n\}$ be an orthonormal subset of Hilbert space H .

a. Prove that for any vector $y \in H$,

$$\|y\|_2^2 \geq \sum_{i=1}^n |\langle y, e_i \rangle|^2$$

b. Propose conditions on y so that equality holds?

2. If $E = \{e_1, e_2, \dots, e_n\}$ and $\tilde{E} = \{\tilde{e}_1, \tilde{e}_2, \dots, \tilde{e}_n\}$ are a pair of biorthogonal bases of a Hilbert space H , prove that for any vector $y \in H$ the following equality holds.

$$\|y\|_2^2 = \sum_i \langle y, e_i \rangle \langle y, \tilde{e}_i \rangle^*$$