Digital Media Laboratory
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Signals & Systems

Adapted from:

- **Lecture notes from MIT**
- **Chapter 2 of M. Vetterli, J. Kovacevic, "Wavelets & Subband Coding", Prentice Hall, 2007**
- **Appendix of S. Mallat, "A Wavelet Tour of Signal Processing" Elsevier 2009**

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SIGNALS

Signals are functions of independent variables that carry information. For example:

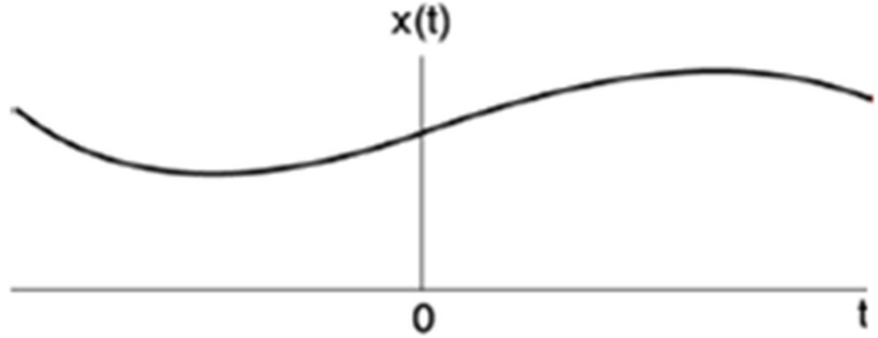
- ✧ Electrical signals --- voltages and currents in a circuit
- ✧ Acoustic signals --- audio or speech signals (analog or digital)
- ✧ Video signals --- intensity variations in an image (e.g. a CAT scan)
- ✧ Biological signals --- sequence of bases in a gene

THE INDEPENDENT VARIABLES

- Can be *continuous*
  - Trajectory of a space shuttle
  - Mass density in a cross-section of a brain
- Can be *discrete*
  - DNA base sequence
  - Digital image pixels
- Can be 1-D, 2-D, ••• N-D
- For this course: Focus on a single (1-D) independent variable which we call “time”.
- Continuous-Time (CT) signals: x(t), t — continuous values

  Discrete-Time (DT) signals: x[n], n — integer values only

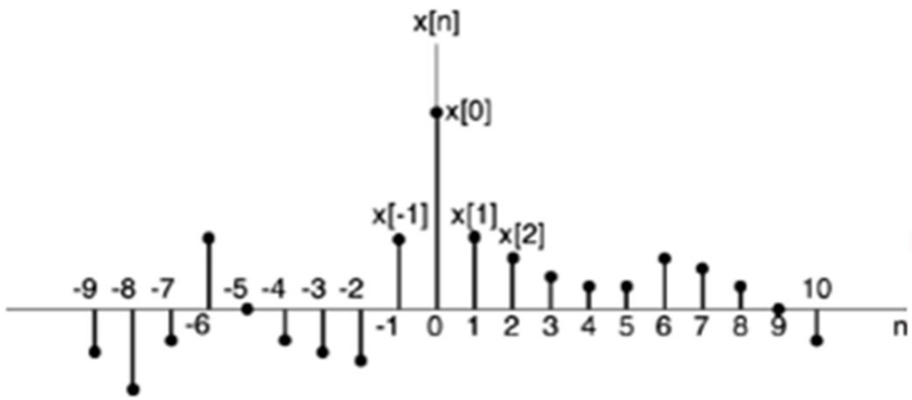
CT Signals



- Most of the signals in the physical world are CT signals—E.g. voltage & current, pressure, temperature, velocity, etc.

DT signals

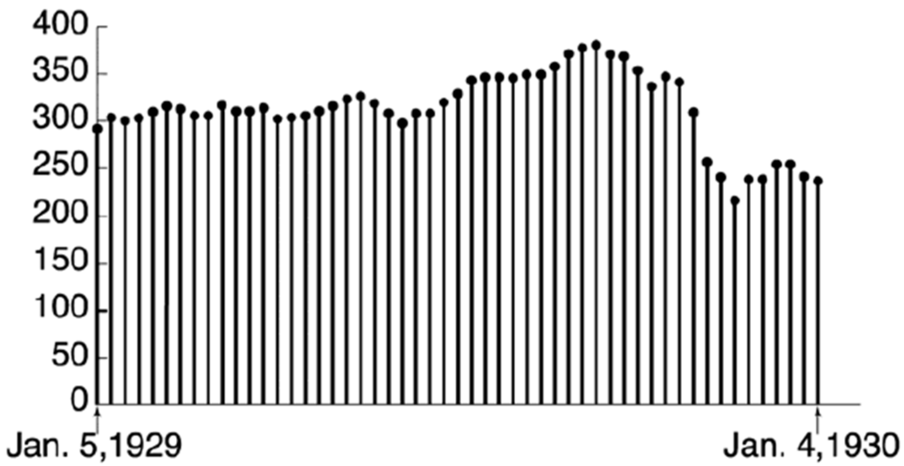
- x[n], n—integer, time varies discretely



- **Examples of DT signals in nature:**
  - DNA base sequence
  - Population of the *nth generation of certain species*•
  - …

Many human-made DT Signals

✧ Example 1: Weekly Dow Jones industrial average



✧ Example 2: digital image

Courtesy of Jason Oppenheim.

✧ Why DT? Can be processed by modern <u>digital</u> computers and <u>digital</u> signal processors (DSPs).

Signal Models (Advanced)

- **Signals are often modeled as members of a vector space**
- **Definition:** A ***vector space*** over the set of complex or real numbers, C or R, is a set vectors, E, together with ***addition*** and ***scalar multiplication***, which, for general x, y in E, and a , b in C or R satisfy the following:
  - Commutativity: x+y = y+x
  - Associativity: (x+y)+z = x+(y+z), a(bx) = (ab)x
  - Distributivity: a(x+y) = ax + ay, (a+b)x = ax + bx
  - Additive Identity: There exists 0 in E, s.t. x + 0 = x for all x in E
  - Additive inverse: for all x in E there exists a (-x) in E s.t x+(-x) = 0
  - Multiplicative identity: 1x = x, for all x in E

Signal Models (Continued)

- **Definition:** An inner product on a vector space E, over C (or R), is a complex valued function <·,·> defined over E×E with the following:

$$\langle x+y,z\rangle=\langle x,z\rangle+\langle y,z\rangle$$

$$\langle ax,y\rangle=a\langle x,y\rangle$$

$$\langle x,y\rangle^{*}=\langle y,x\rangle$$

$$\langle x,x\rangle\geq 0$$

$$\langle x,x\rangle=0 \;\; iff \;\; x=0$$

- Examples: Standard inner products for complex valued functions over C and Z:

$$\langle f,g\rangle=\int_{-\infty}^{+\infty} f(t)g^{*}(t)dt$$

$$\langle x,y\rangle=\sum_{n=-\infty}^{+\infty} x[n]y^{*}[n]$$

Signal Models (Continued)

- Two vectors x, y are said to be ***orthogonal*** if <x,y>=0
- We normally work within a vector space H that admits a norm. A ***norm*** satisfies the following properties:

$$\forall f \in H, \quad \|f\| \geq 0 \quad and \quad \|f\| = 0 \Leftrightarrow f = 0$$
$$\forall \lambda \in \mathbb{C}, \quad \|\lambda f\| = |\lambda| \|f\|$$
$$\forall f, g \in H, \quad \|f + g\| \leq \|f\| + \|g\|$$

- The space $L^p(\mathbb{R})$ ($l^p(\mathbb{R})$) is composed of functions f on R (Z) for which:

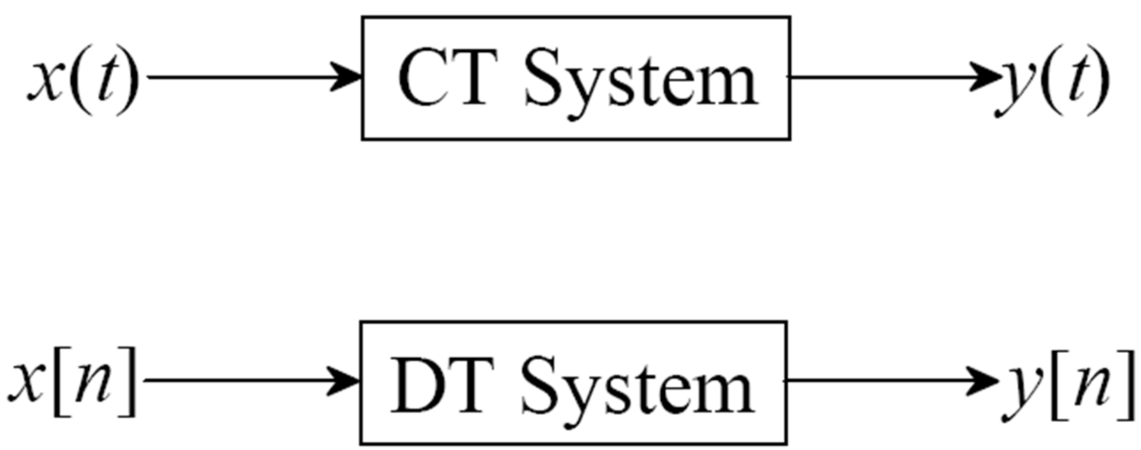
$$\|f\|_p = \left(\int_{-\infty}^{+\infty} |f(t)|^p \, dt \right)^{1/p} < +\infty, \quad \|f\|_p = \left(\sum_{n=-\infty}^{+\infty} |f[n]|^p \right)^{1/p} < +\infty$$

Signal Models (Continued)

- **Definition:** A sequence $\{f_n\}_{n\in\mathbb{N}}$ is a ***Cauchy sequence*** if for any $\varepsilon > 0$, if n and p are large enough, then $\|f_p - f_n\| < \varepsilon$.
- The space H is said to be ***complete*** if every Cauchy sequence in H converges to an element of H.
- A ***Banach space*** is a vector space that admits a norm and is also complete.
- A ***Hilbert*** space is a ***Banach*** space with an inner product.
- **In this course we are mainly interested in signals that belong to a well defined *Hilbert space*.**

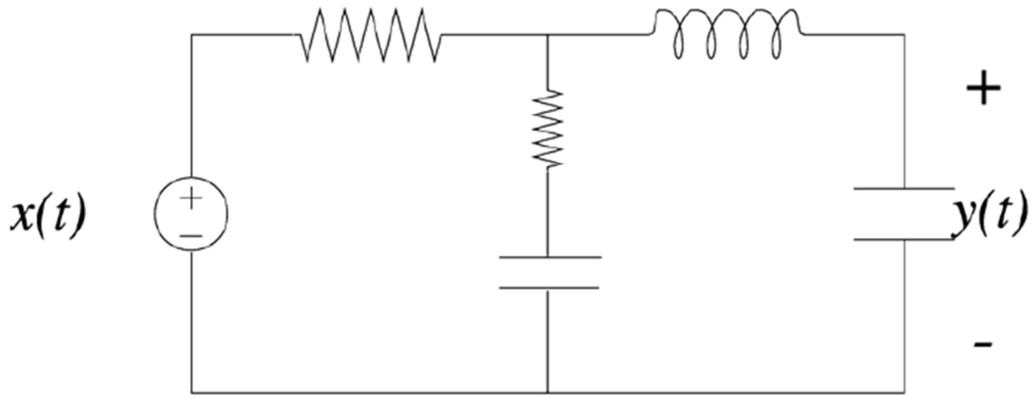
SYSYEMS

- **SYSTEMS- For the most part, our view of systems will be from an input-output perspective:**
  - **A system responds to applied input signals, and its response is described in terms of one or more output signals**



EXAMPLES OF SYSTEMS

- An RLC circuit



- Dynamics of an aircraft or space vehicle
- An algorithm for analyzing financial and economic factors to predict bond prices
- An algorithm for post-flight analysis of a space launch
- An edge detection algorithm for medical images EXAMPLES OF SYSTEMS

SYSTEM INTERCONNECTIOINS

- **An important concept is that of interconnecting systems**
  - **To build more complex systems by interconnecting simpler subsystems**
  - **To modify response of a system**
- **Signal flow (Block)**