Signals and Systems September 20, 2012
Sharif University of Technology CE 40-242
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Date Due: Azar 10, 1391

Homework 7

Solutions:

1. Determine the Laplace transforms (including the regions of convergence) of each of the following signals.

   Laplace Transform of $x(t) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$

   a. $X(s) = \frac{e^{-3s}}{s+2}, \quad ROC : Real(s) > -2$

   b. $X(s) = \frac{1}{s} - \frac{1}{s+3} + \frac{1}{(s+3)^2}, \quad ROC : Real(s) > -3$

   c. $X(s) = \frac{1}{(s-1)^2} + \frac{1}{(s+1)^2}, \quad ROC : -1 < Real(s) < 1$

   d. $X(s) = \frac{1}{s^2}(1 - e^{-s} - e^{-2s} + e^{-3s}), ROC : R$

2. Determine all possible signals with Laplace transforms of the following forms. For each signal, indicate the associated region of convergence.

   a. $x(t) = e^{-t}u(t) + te^{-t}u(t), \quad ROC : Real(s) > -1$

   b. $X(s) = \frac{1}{s+1} + \frac{-1}{s} + \frac{1}{s^2}$
   $ROC : Real(s) > 0, x(t) = tu(t) - u(t) + e^{-t}u(t)$
   $ROC : -1 < Real(s) < 0(s), x(t) = -tu(-t) + u(-t) + e^{-t}u(t)$
   $ROC : Real(s) < -1, x(t) = -tu(-t) + u(-t) - e^{-t}u(-t)$

   c. $X(s) = \frac{0.5}{s+1+i} + \frac{0.5}{s+1-i}$
   $ROC : Real(s) > 1, \quad x(t) = \frac{cos(t)}{e^t}u(t)$
   $ROC : Real(s) < 1, \quad x(t) = \frac{cos(t)}{e^t}u(-t)$

   d. $ROC : Real(s) > 0, \quad x(t) = tu(t) - 2(t-1)u(t-1) + (t-2)u(t-2)$ $ROC : Real(s) < 0, \quad x(t) = -tu(-t) - 2(1-t)u(1-t) + (2-t)u(2-t)$

3. **a.** $H(s) = \frac{s-1}{s+1}$
   step response of system: $2e^{-t}u(t) - u(t)$

   b. $y(t) = e^{-t}u(t) - 2te^{-t}u(t)$

4. **a.** $x(0) = 1$
   $x(\infty) = 0$

   b. $x(0) = 0$
   $x(\infty) = 1$

   c. $x(0) = 0$
   $x(\infty) = 1$

d. $x(0) = 0$
$x(\infty) = +\infty$

e. $x(0) = 0$
$x(\infty) = 0$

f. $x(0) = 0$
$x(\infty) = 0$

5. $X(s)H_3(s)(H_1(s) + H_2(s)) = Y(s)(1 + H_4(s))(H_1(s) + H_2(s))$
$H(s) = \frac{H_3(s)H_1(s)+H_3(s)H_2(s)}{1+H_4(s)H_1(s)+H_4(s)H_2(s)}$

6. **a.** Because both F and G are casual systems, ROC of F is Real(s)¿0 and ROC of G is Real(s)¿1, So the ROC of neither F nor G includes the iw axis, so F and G are not stable systems.

   b. We have a MISO (multiple-input single-output) system. Using superposition, we can find the transfer function from X $\rightarrow$ Y and V $\rightarrow$ Y . Start with X $\rightarrow$ Y :
In this case we have:
with zero V we have:$H_x = \frac{Y}{X} = \frac{FG}{1+FG}$
For V $\rightarrow$ Y and zero X we have: $H_v = \frac{Y}{V} = \frac{G}{1+FG}$
Notice the denominators in above statements are the same, meaning the transfer functions have the same poles. Now
$1 + FG = \frac{s(s1)+K(s+2)}{s(s-1)}$
The function of interest is $s(s1) + K(s + 2) = s2 + (K1)s + 2K$. The roots are:
$\frac{(K-1)\pm\sqrt{(K-1)^2-8K}}{2}$
The last equation describes the poles of the transfer function. We must consider two cases: when the poles are real, and when theyre complex. In order for the poles to be real, we must have $(K1)^2 8K \geq 0$, which occurs when $K \leq 52\sqrt{6}$, or $K \geq 5 + 2\sqrt{6}$. In this case, for stability, the poles must be negative, which requires that $K1 > \sqrt{(K-1)^2 - 8K}$, and only $K \geq 5+2\sqrt{6}$ will satisfy this requirement. When the poles are complex (i.e., $(K1)^2 8K < 0$, or $52\sqrt{6} < K < 5 + 2\sqrt{6}$), the requirement for stability is that the real part of the poles is negative, which implies $K > 1$.Combining these two cases, we find that $K > 1$ will yield a stable system.

Practical Assignment:

I. The answer of this section is in ”HW7 practical sol” file.