Signals and Systems
Sharif University of Technology
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CE 40-242

Date Due: Mehr 17, 1391

Homework 2 (Chapters 1 & 2)

Problems

1. Determine and sketch the following signal and its even and odd parts. Label your sketches carefully.
   $x(t) = u(t+1) - u(t) + (u(t) - u(t-1)) * (u(t) - u(t-1)) * \delta(t-1)$

2. Are the following signals periodic? If so determine their fundamental period.

   a. $x(t) = \cos(6t) + \sin(5\pi t) + 2\sin^2(3t)$

   b. $y(t) = \frac{1}{1+\sin^2(\pi t)} + 2\cos(2\pi t)$

   c. $w(t) = \cos(\frac{\pi}{3}t) + \cos(\frac{2\pi}{5}t)$

3. For each one of the statements below, Provide a proof if you believe the statement is true. Provide a counterexample if you believe the statement is false.

   a If a discrete-time system is BIBO stable, causal and linear, then it must be time-invariant.

   b If a continuous-time system is memoryless, causal and time-invariant, then it must be linear.

4. A system may or may not be linear, time-invariant, memoryless, causal, or stable. Determine whether or not each of the following systems has these properties.

   a. $y(t) = \frac{1}{2}^{|t|} x(t)$

   b. $y(t) = x(t+3) - x(1-t)$

   c. $y[n] = min_{0 \le m \le n}[x[m]]$

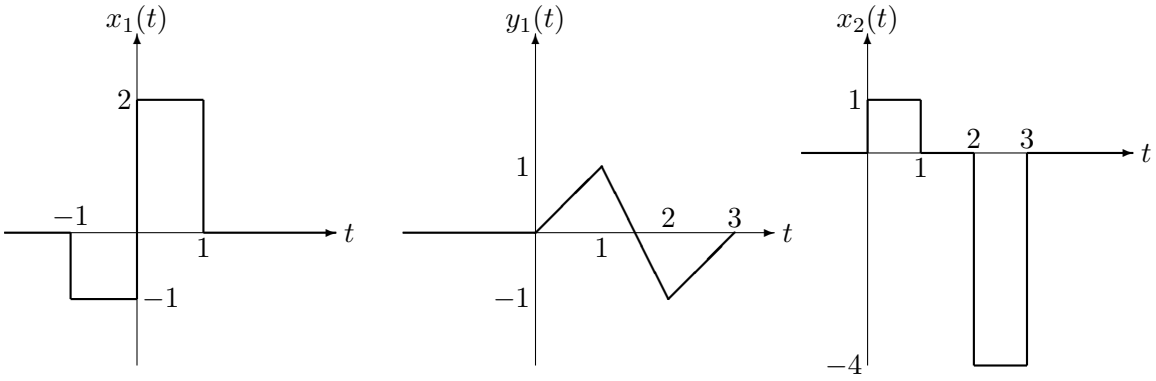
   d. $y[n] = \sum_{m=-\infty}^{n+1} x[m]$

5. Suppose that a causal, Discrete-time system F has the input-output signal pair x and y described below:

   $$x[n] = 2\delta[n-1] \longrightarrow y[n] = \delta[n+1] + \delta[n-1]$$

   Based on this information, is F (i) linear and (ii) time-invariant? Explain your choice clearly.

6. Consider an LTI system whose response to the signal $x_1(t)$ is the signal $y_1(t)$ where these signals are depicted below. Determine and provide a labeled sketch of the response to the input $x_2(t)$, which is also depicted below.



7. Let $x[n], y[n]$ denote discrete-time signals. Prove the following property of convolution sum:

$$(x * y)[n - \eta] = x[n - \eta] * y[n] = x[n] * y[n - \eta]$$

8. Compute the convolution sum $y[n] = x[n] * h[n]$ for each of the following pairs of signals. (P 2.21 b,c p. 141)

   a. $x[n] = h[n] = \alpha^n u[n]$

   b. $x[n] = (-\frac{1}{2})^n u[n-4]$, $h[n] = 4^n u[2-n]$

9. Compute the convolution $y(t) = x(t) * h(t)$ for each of the following pairs of signals.

   a. $x(t) = e^{-t}u(t+1), h(t) = e^{2t}u(-t)$

   b. $x(t) = x_1(t)$ (from Prob. 6), $h(t) = (u(t) - u(t-1)) * (u(t) - u(t-1))$

10. The following are the impulse responses of LTI systems. Determine whether each system is causal and/or stable. Justify your answers. (P 2.28 e,g and 2.29 e,f)

    a. $h[n] = [1 - 0.99^n]u[n]$

    b. $h[n] = 2^n u[3-n]$

    d. $h(t) = te^{-t}u(t)$

    e. $h(t) = u(1-t) - \frac{1}{2}e^{-t}u(t)$

11. P 2.40 p. 148

12. P 2.47 p. 152