Date Due: Aban 15, 1391

Homework 4 (Chapter 4)

Problems

1. Derive the following formulas in your own word. You will get a zero for the whole homework if you copy the text from your book. Explain each line of the proof.

   a. $x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{-j\omega t}\,\mathrm{d}\omega$

   b. $X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,\mathrm{dt}$

2. Suppose that real signal $x(t)$ has the Fourier Transform $X(j\omega)$. Suppose another signal $y(t)$ has the same shape as $X(j\omega)$.

   a. Determine $Y(jw)$ Fourier Transform in terms of $X(j\omega)$.

   b. Using the result of the last part show that:

   $$F\{e^{-jAt}\} = 2\pi\delta(\omega + A)$$

3. Determine the continuous-time signal corresponding to each of the following transforms:

   a. $X(j\omega) = j[\delta(\omega+2) + \delta(\omega-1) * j] - [\delta(\omega+1) + \delta(\omega-2) * j]$

   b. $X(j\omega) = 2\cos(4\omega - \pi/2)$

4. Compute the Fourier transform of each of the following signals:

   a. $[e^{-\alpha t}\cos 2\omega_0 t]u(t), \alpha > 0$

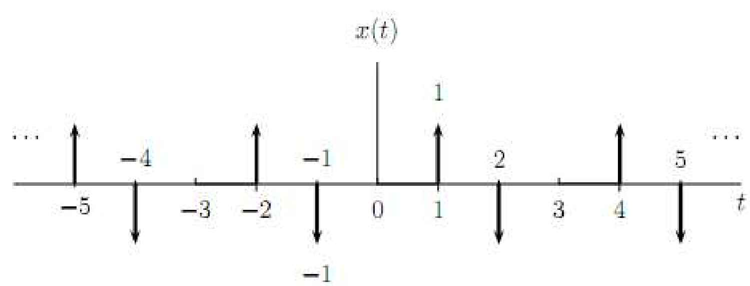
   b. $x(t) = e^{-|2t|}\sin 2t$

   c. $x(t) = \begin{cases} 1 + \sin \pi t, & |t| \leq 2 \\ 0, & |t| > 2 \end{cases}$

   d. $[\frac{\sin \pi t}{\pi(t-1)}][\frac{\sin \pi(t-1)}{\pi t}]$

   e. $x(t) = \begin{cases} 2 + t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

   g. The signal $x(t)$ depicted below:



5. Determine which, if any, of the real signals depicated in below have Fourier transforms that satisfy each of the following conditions:

   a. $\mathcal{R}e\{X(j\omega)\} + \mathcal{I}m\{X(j\omega)\} = 0$

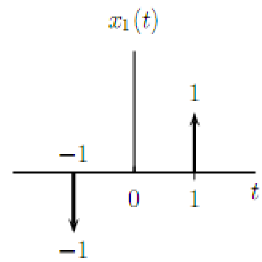
   b. There exists a real α such that $e^{j\alpha\omega}X(j\omega)$ is real

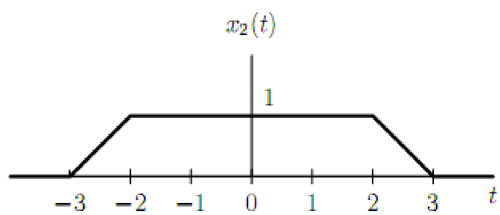
   c. $X(j0) = 0$

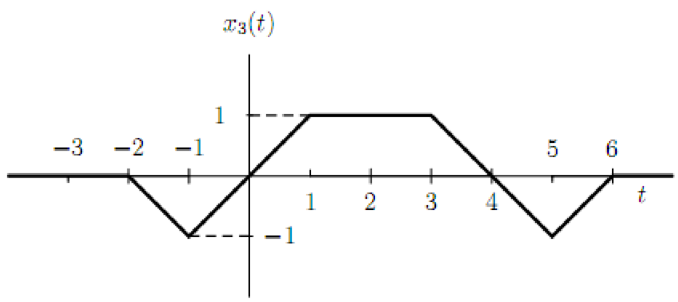
   d. $\int_{-\infty}^{\infty} X(j\omega)d\omega = 0$

   e. $\int_{-\infty}^{\infty} \omega X(j\omega)d\omega = 0$

   f. $X(j\omega)$ is not periodic







6. Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ depicted in Figure P4.25 of textbook p. 341. (P 4.25 p. 341 with little change) ***Note:*** You should perform all these calculations without explicitly evaluating $X(j\omega)$.

   a. $X(j\omega)$ can be written as $A(j\omega)e^{j\theta(j\omega)}$ where $A(j\omega)$ and $\theta(h\omega)$ are real. Find $\theta(j\omega)$.

   b. Find $e^{-j}X(j0)$.

   c. Find $\int_{-\infty}^{\infty} X(j\omega)d\omega$.

   d. Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$.

e. Sketch the inverse Fourier transform of $\mathcal{R}e\{X(j\omega)\}$ and $\mathcal{I}m\{X(j\omega)\}$.

7. The output of a **casual** LTI system is related to the input $x(t)$ by the equation:

$$\frac{dy}{dt} + \frac{d^2y}{dt^2} = \int_{-\infty}^{\infty} x(\tau)z(t-\tau)d\tau + x(t)$$

where $z(t) = e^{-2t}u(t) + \delta(t)$
determin the impulse response of the system.

8. Let $g_1(t) = \{[\cos(\omega_0 t)]x(t)\} * h(t)$ and $g_2(t) = \{[\sin(\omega_0 t)]x(t)\} * h(t)$ where

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk100t}$$

is a real-valued periodic signal that $h(t)$ is the impluse response of a stable LTI system.

a. Specify a value for ω_0 and any necessary constraints on $H(j\omega)$ to ensure that

$$g_1(t) = \mathcal{R}e\{a_5\} \text{ and } g_2(t) = \mathcal{I}m\{a_5\}.$$

b. Give an example of $h(t)$ such that $H(j\omega)$ satisfies the constraints you specified in part (a). (P4.42 p. 348)

Practical Assignment

1. Consider a discrete-time system H_1 with impulse response

$$h_1[n] = u[n] - u[n-1] + u[n-2] - u[n-5],$$

a discrete-time system H_2 with impulse response

$$h_2[n] = (\frac{1}{3})^n (u[n+4] - u[n-4]),$$

and a discrete-time signal

$$x[n] = (\frac{1}{2})^n (u[n-1] - u[n-5]).$$

The signals $h_1[n]$, $h_2[n]$, and $x[n]$ are all defined for $-10 \leq n \leq 10$.

a. Plot $h_1[n]$, $h_2[n]$, and $x[n]$ together using the *subplot* function.

b. Consider a system H formed from the series connection of H_1 and H_2, where $x[n]$ is input to H_1, the output $v[n]$ of H_1 is input to H_2, and the output of H_2 is $y[n]$. Use the *conv* function to find $v[n]$ and $y[n]$. Plot $v[n]$ and $y[n]$ using the *subplot* function.

c. Now assume that the order of the systems is reversed, so that $x[n]$ is input to H_2, the output $v[n]$ of H_2 is input to H_1, and $y[n]$ is the output of H_1. Plot $v[n]$ and $y[n]$. Briefly explain why $v[n]$ is different in parts (b) and (c), whereas $y[n]$ is the same in both parts.