Signals and Systems
Sharif University of Technology
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CE 40-242

Date Due: Azar 6th, 1391

Homework 5
(Chapter 5 - DFT)

Problems

1. Computing the Fourier transform.
   - 5-21. a
   - 5.21. b
   - 5.21. d
   - 5.21. f
   - 5.21. h
   - 5.21. j
2. Determining corresponding signals of the transforms.
   - 5-22. a
   - 5-22. b
   - 5-22. d
   - 5-22. e
   - 5-22. f
   - 5-22. h
3. $x[n]$ is a finite-duration signal of length N so that $x[n] = 0, n < 0$ and $n > N-1$. The discrete-time Fourier transform of x[n] is denoted by $X[\Omega]$. We generate the periodic signal $\tilde{y}[n]$ by periodically replicating x[n], i.e.,

$$\tilde{y}[n] = \sum_{r=-\infty}^{\infty} x[n+rN]$$

   (a) Write the expression in terms of $x[n]$ for the Fourier series coefficients a_k of $\tilde{y}[n]$.

   (b) Write an expression relating the Fourier series coefficients of $\tilde{y}[n]$ to the Fourier transform of $x[n]$.

4. Suppose we are given the following facts about an LTI system S with impulse response $h[n]$ and frequency response $H(e^{j\omega})$. Determine $h[n]$.

   (a) $(\frac{1}{4})^n u[n] \longrightarrow g[n]$, where $g[n] = 0$ for $n \geq 2$ and $n < 0$.

   (b) $H(e^{j\pi/2}) = 1$.

(c) $H(e^{j\omega}) = H(e^{j(\omega-\pi)})$.

5. Consider a system consisting of the cascade of two LTI systems described by the difference equation:

$$y[n] = x[n] - x[n-1] + 3x[n-4]$$

and

$$y[n] - \frac{1}{3}y[n-1] = x[n]$$

(a) Find the difference equation describing the overall system.

(b) Determine the impulse response of the overall system.

(c) If $g[n]$ is response of system to input x[n], find system response to $2x[n-1]$ in terms of $g[n]$.

6. Let the inverse Fourier transform of $Y(e^{j\omega})$ be

$$y[n] = \left(\frac{sin(\omega_c n)}{\pi n}\right)^2,$$

where $0 < \omega_c < \pi$. Determine the value of ω_c wich ensures that

$$Y(e^{j\pi}) = \frac{1}{2}$$

.

7. Compute the DFT of each of the following finite-length sequences considered to be of length N (where N is even):

(a) $x[n] = \delta[n]$

(b) $x[n] = \delta[n - n_0], \qquad 0 \leq n_0 \leq N-1,$

(c) $x[n] = \begin{cases} 1, & n\ even, \quad 0 \leq n \leq N-1, \\ 0, & n\ odd, \quad 0 \leq n \leq N-1, \end{cases}$

(d) $x[n] = \begin{cases} 1, & 0 \leq n \leq N/2 - 1, \\ 0, & N/2 \leq n \leq N-1, \end{cases}$

(e) $x[n] = \begin{cases} a^n, & 0 \leq n \leq N-1, \\ 0, & otherwise. \end{cases}$

8. Consider a 20-point finite-duration sequence $x[n]$ such that $x[n] = 0$ outside $0 \leq n \leq 19$, and let $X(e^{j\omega})$ represent the Fourier transform of $x[n]$.

(a) If it is desired to evaluate $X(e^{j\omega})$ at $\omega = 4\pi/5$ by computing one M-point DFT, determine the smallest possible M. and develop a method to obtain $X(e^{j\omega})$ at $\omega = 4\pi/5$ using the smallest M.

(b) If it is desired to evaluate $X(e^{j\omega})$ at $\omega = 10\pi/27$ by computing one L-point DFT, determine the smallest possible L. and develop a method to obtain $X(e^{j10\pi/27})$ using the smallest L.

Practical Problems

1. write a MATLAB function to compute the DTFT of a finite-duration sequence. The format of the function should be

   function [X] = dtft(x, n, w)
   % [X] = dtft(x, n, w)
   % X = DTFT values computed at w frequencies
   % x = finite duration sequence over n
   % n = sample position vector
   % w = frequency location vector

2. For a linear, shift-invariant system described be difference equation

$$y(n) = \sum_{m=0}^{M} b_m x(n-m) - \sum_{l=1}^{N} a_l y(n-l)$$

   the frequency-response function is given by

$$H(e^{j\omega}) = \frac{\sum_{m=0}^{M} b_m e^{-j\omega m}}{1 + \sum_{l=1}^{N} a_l e^{-j\omega l}}$$

   Write a MATLAB function *freqresp* to implement this relation. The format of this function should be

   function [H] = freqresp(b, a, w)
   % [H] = freqresp(b, a, w)
   % H = frequency response array evaluated at w frequencies
   % b = numerator coefficient array
   % a = denominator coefficient array [a(1) = 1]
   % w = frequency location array