Signals and Systems November 20, 2012
Sharif University of Technology CE 40-242
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Date Due: Azar 13, 1391

Homework 6 (Chapter 7)

Problems

1. Figure P7.28(a) (on page 566 of textbook) shows a system that converts a continuos-time signal to a discrete-time signal. The input $x(t)$ is periodic with a period of 0.1 second. The fourier series coefficients of $x(t)$ are

$$a_k = (\frac{1}{2})^{|k|}, -\infty < k < +\infty$$

The lowpass filter $H(j\omega)$ has the frequency response shown in Figure p7.28(b) (on page 566 of textbook). The sampling period $T = 5 \times 10^{-3}$ second. (P 7.28 p. 586)

a. Show that $x[n]$ is periodic sequence, and determine its period.

b. Determine the Fourier series coefficents of $x[n]$.

2. Let $x_c(t)$ be a continuous-time signal whose Fourier transform has the property that $X_c(j\omega) = 0$ for $|\omega| \geq 2000\pi$. A discrete-time signal

$$x_d[n] = x_c(n(0.5 \times 10^{-3}))$$

is obtained. For each of the following constraints on the Fourier transform $X_d(e^{j\omega})$ of $x_d[n]$, determine the corresponding constraint on $X_c(j\omega)$

1. $X_d(e^{j\omega})$ is real.

2. The maximum value of $X_d(e^{j\omega})$ over all ω is 1 .

3. $X_d(e^{j\omega}) = 0$ for $\frac{3\pi}{4} \leq |\omega| \leq \pi$.

4. $X_d(e^{j\omega}) = X_d(e^{j(\omega-\pi)})$.

3. The following facts are given about the signal $x[n]$ and its Fourier transform:

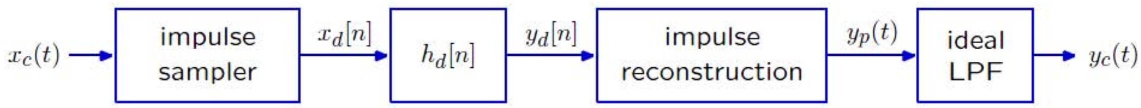
1. $x[n]$ is real.

2. $X(e^{j\omega}) \neq 0$ for $0 < \omega < \pi$.

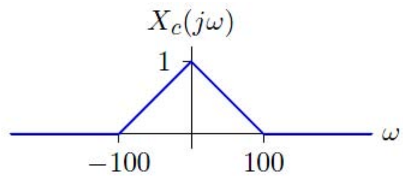
3. $x[n] \sum_{k=-\infty}^{+\infty} \delta[n - 2k] = \delta[n]$.

Determine $x[n]$. You may find it useful to note that the signal $(\frac{sin \frac{\pi n}{2}}{(\pi n))}$ satisfies two of these conditions.

4. Sampling and its reconstruction allow us to process CT signals using digital electronics as shown in the following figure.



The "impulse sampler" and "impulse reconstruction" use sampling interval $T = \frac{\pi}{100}$. The unit-sample function $h_d[n]$ represents the unit-sample response an ideal DT lowpass filter with gain 1 for frequencies in the range $\frac{-\pi}{2} < \Omega < \frac{\pi}{2}$. The "ideal LPF" passes frequencies in the range $-100 < \omega < 100$. It also has a gain of T thoroughout its pass band.
Assume that the Fourier transform of the input $x_c(t)$ is $X(j\omega)$ shown below.



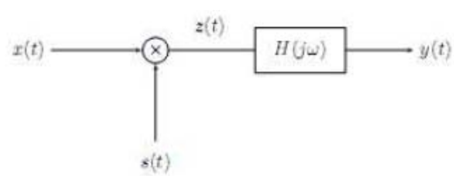
Determine $Y_c(j\omega)$.

5. A signal $x[n]$ has a Fourier transform $X(e^{j\omega})$ that is zero for $\frac{\pi}{4} \leq |\omega| \leq \pi$.
Another signal

$$g[n] = x[n] \sum_{k=-\infty}^{+\infty} \delta[n-1-4k]$$

is generated.Specify the frequency response $H(e^{j\omega})$ of a lowpass filter that produces $x[n]$ as output when $g[n]$ is the input.

6. A sinusoidal signal $x(t) = cos(10t)$ is sampled and filtered as shown below.

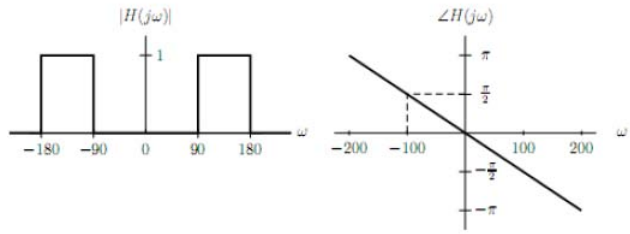


the frequnecy response of the filter is

$$|H(j\omega)| = \begin{cases} 1 & 90 < \omega < 180 \\ 0 & otherwise. \end{cases}$$

$$\langle H(j\omega) = -\frac{\pi\omega}{200}$$

as shown in the figure below :



a. Suppose $s(t) = \sum_{k=-\infty}^{+\infty} \delta(t - KT)$ and $T = \frac{2\pi}{90}$. Provide a labeled sketch of $Z(jw)$, the Fourier transform of $z(t)$.

b. find $y(t)$, assumming the $s(t)$ and the value of T given in part (a).

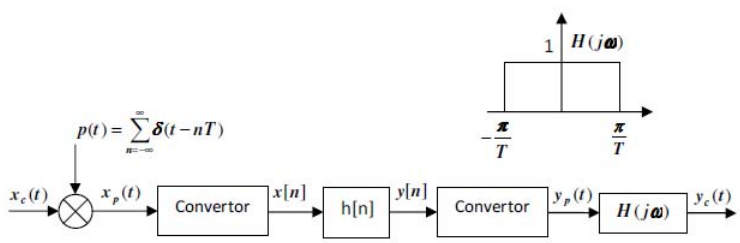
Practical Assignment

1. Plot $y[n]$ and $y_c(t)$ of the system figure below for the given input signals for $T = 0.05, 0.125, 0.25$. Draw your figure for the time interval $-3 \leq t \leq 3$.

   a. $x_1(t) = sin(10\pi t)$

   b. $x_2(t) = sin(6\pi t) + 3cos(15\pi t)$

   c. $x_3(t) = sin(6\pi t) + 3cos(15\pi t) + noise$ (For creating noise signal use **rand** function).



$$H(e^{j\omega}) = \begin{cases} 1 & , |\omega| \leq \frac{2\pi}{3} \\ 0 & , \frac{2\pi}{3} < |\omega| \leq \rho \end{cases}$$