Signals and Systems Final exam 2009/06/23

1. < Find the FT/DTFT >
   (a) (5%) $x(t) = |\sin(\pi t/2)|$, please find the FT representation of it.
   (b) (5%) $x[n]$ as depicted in Fig.1b, please find the DTFT representation of it.

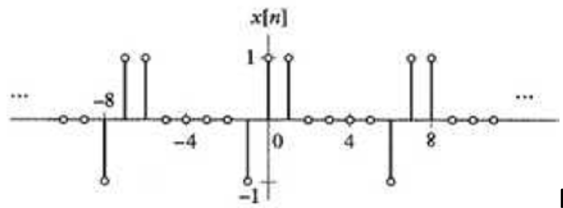


Fig.1b

2. < Mixtures of periodic/nonperiodic and continuous/discrete-time signals >
   (a) (5%) Consider the system depicted in Fig.2a. The impulse response is given by

   $h(t)=\dfrac{\sin(13\pi t)}{\pi t}$, and we have $x(t)=\sum_{k=1}^{\infty}\dfrac{1}{(k+1)^2}\cos(k4\pi t)$, and $g(t)=\sum_{k=0}^{3}\cos(k7\pi t)$.

   Use the FT to determine y(t).
   (b) (5%) Consider the discrete-time system depicted in Fig.2b.

   Use the DTFT to determine the output y[n] for the following cases:

   Let $h[n]=\delta[n]+\dfrac{\sin(\frac{\pi}{3}n)}{\pi n}-\dfrac{\sin(\frac{3\pi}{4}n)}{\pi n}$, $x[n]=2-\sin(\frac{2\pi}{15}n)+\cos(\frac{5\pi}{6}n)$, $w[n]=(-1)^{n+1}$

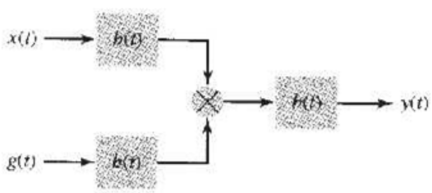


Fig.2a

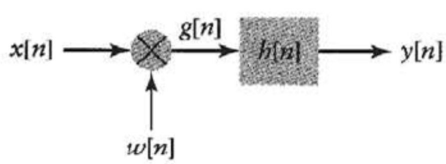
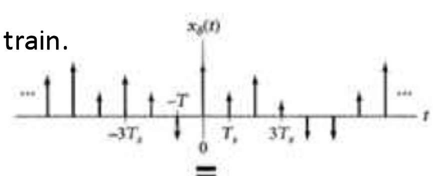
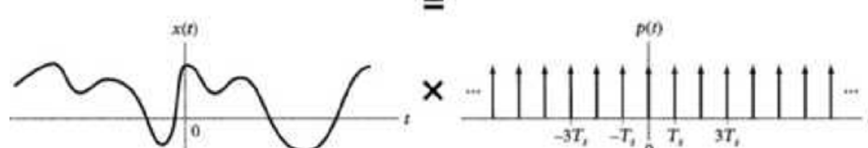


Fig.2b

3. < Sampling >
   (a) (4%) Consider discrete-time signal x[n] which is equal to the "samples" of continuous-time signal x(t). Define $x_\delta(t)=\sum_{n=-\infty}^{\infty}x[n]\delta(t-nT_s)$ which is sampled from x(t) and

   $p(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$ as an impulse train.

   Find the FT of $x_\delta(t)$ with sampling frequency $\omega_s=2\pi/T_s$
   (b) (4%) Consider the effect of sampling the sinusoidal signal $x(t)=\cos(\pi t/2)$.

   Determine the FT of the sampled signal for (i)$T_s=\frac{1}{8}$, and (ii) $T_s=\frac{3}{4}$.
   (c) (4%) Consider subsampling the signal x[n] so that y[n]=x[qn].

   Sketch $Y(e^{j\Omega})$ with $x[n]=\dfrac{\sin(\frac{\pi n}{5})}{\pi n}$ and q=3.

4. < Reconstruction >

(a) (5%) x(t) = a(t)b(t), is sampled with sampling interval T_s, determine the bounds on T_s, which guarantee that there is no aliasing. Where the FT's A(jω) and B(jω) are depicted in Fig.4a.

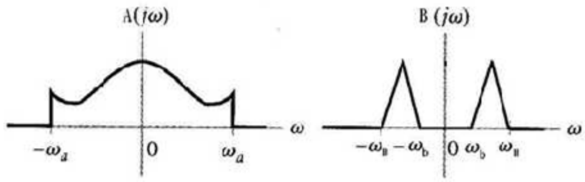


Fig.4a

(b) (5%) Consider the system depicted in Fig.4b, and X(jω), w(t) are also given in Fig.4b.

Find the largest value of T such that x(t) can be reconstructed from y(t).

Determine a system that will perform the reconstruction for this maximum value of T.

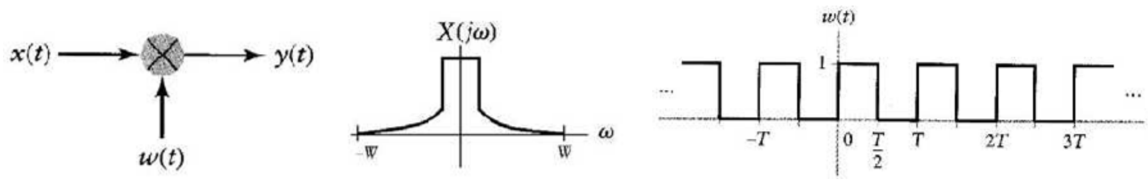


Fig.4b

5. Fig.5 shows the FT of x(t).

Please match the following descriptions with the figures below.

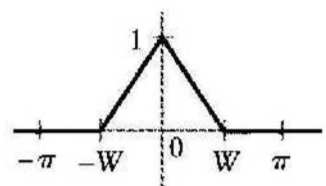


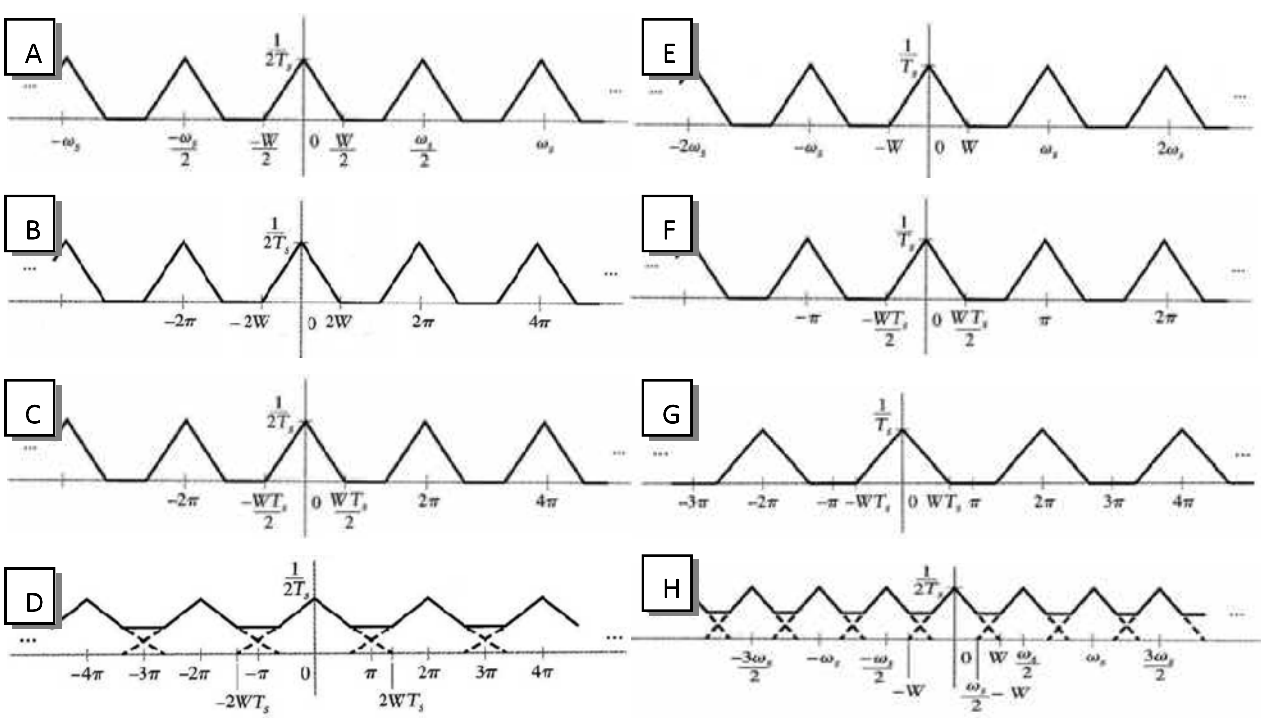
Fig.5

(a) (2%) Find $X_\delta(j\omega)$, where $x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$.

(b) (2%) Find $X'_\delta(j\omega)$, where $T_s' = 2T_s$.

(c) (2%) Find $X(e^{j\Omega})$, where $x[n] = x(nT_s)$.

(d) (2%) Find $Y(e^{j\Omega})$, where $y[n] = x[2n]$.   Let $\omega_s = \frac{2\pi}{T_s}$, $2W \le \omega_s \le 3W$, $W > \frac{\pi}{2}$



6.(15%) Use the unilateral Laplace transform to determine the output of a system represented by the differential equation

$$\frac{d^2}{dt^2}y(t)+5\frac{d}{dt}y(t)+6y(t)=\frac{d}{dt}x(t)+5x(t)$$

in response to the input x(t)= e^{-t} u(t).

Assume that the initial conditions on the system are $y(0^-)$=1 and $\left.\frac{d}{dt}y(t)\right|_{t=0^-}$ =3

Identify the forced response of the system $y^{(f)}(t)$ **and the natural response** $y^{(n)}(t)$

7.(20%) Use the method of partial fractions to determine the time signals corresponding to the following bilateral Laplace transforms:

$$X(s)=\frac{3s^2+10s+10}{(s+2)(s^2+6s+10)}$$

(i) with ROC Re(s)<-3

(ii) with ROC Re(s)>-2

(iii) with ROC -3<Re(s)<-2

(iv) if the system is causal ,find the impulse response.

(v) if the system is stable ,find the impulse response.

8.(10%) **Sketch the Bode diagrams** for the systems described by the following transfer function:

H(s)= $\frac{6}{(s+1)^3}$ Hint: log6=0.77

9.(10%) Given the Z-transform pair $n^3 7^n u[n] \leftrightarrow X(z)$

Use the Z-transform properties to determine the time-domain signals corresponding to the following Z-transform

[illegible] z^{-1} [illegible]

[illegible] $\frac{d}{dz}X(z)$ [illegible]

10.(15%) **Draw block diagram implementation** of the following system as cascade of second-order sections with real-valued coefficients. And **depict the cascade form for this system.**

$$H(z)=\frac{(1+3z^{-1})^2(1-\frac{1}{2}e^{j\frac{\pi}{3}}z^{-1})(1-\frac{1}{2}e^{-j\frac{\pi}{3}}z^{-1})}{(1-\frac{1}{2}e^{j\frac{\pi}{5}}z^{-1})(1-\frac{1}{2}e^{-j\frac{\pi}{5}}z^{-1})(1-\frac{5}{8}e^{j\frac{\pi}{4}}z^{-1})(1-\frac{5}{8}e^{-j\frac{\pi}{4}}z^{-1})}$$