Signals and Systems

Sharif University of Technology

Date Due: Esfand 13, 1390

Homework 2 (Chapters 1 & 2)

Problems:

1. Consider the signal

$$x(t) = u(t) - u(t-1) + (4-2t)[u(t-1) - u(t-2)]$$

   a. Sketch the signal x(t).
   b. Sketch the odd part of x(t).
   c. Sketch the signal $y(t) = 2x(3 - \frac{t}{2})$.
   d. Sketch the signal $g(t) = \frac{d}{dt}x(t)$

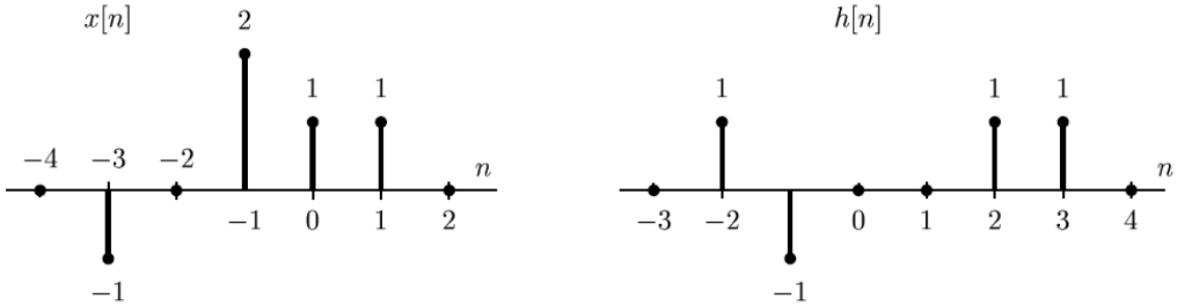
2. Determine whether each of the following signals is periodic. If the signal is periodic, state its period and fundamental frequency.
   a. $[\sin(\pi n/5)]/(\pi n)$
   b. $e^{j(2\pi n/8)}$
   c. e^{jn}

3. For each of following systems determine whether the system is stable, causal, linear, time invariant and memory less:
   a. $y[n] = x[-|n|]^2$
   b. $y[n] = x[n] + x[n-1]$
   c. $y[n] = (-0.5)^2(x[n]+1)$
   d. $y[n] = x[n^2]$
   e. $y[n] = (\cos(\pi n)).x[n]$
   f. $y[n] = \sum_{k=n-1}^{\infty} x[k]$
   g. $y(t) = e^{-|x(t)|}.\log(x(t) + 1)$
   h. $y(t) = x(\frac{t}{2}).\cos(x^2(t))$

4. Consider a system with input x[n] and output y[n]. The input-output relationship is defined by the following two properties:
   a. $y[n] + \alpha y[n-1] = x[n]$,
   b. $y[0] = 1$.

   a. Determine whether the system is time invariant.
   b. Determine whether the system is linear.
   c. Assume that the difference equation (property 1) remains the same, but the value y[0] is specified to be zero. Does this change your answer to either Part (a) or Part (b)?

5. Compute the convolution sum y[n] = x[n]*h[n] for each of the following pairs of signals.



   a. x[n] = u[n+4] – u[n-1] , h[n] =2^n .u[2-n]
   b. x[n] and h[n] are depicted above.

6. Prove that if a system is linear and causal, then is in initial rest condition (i.e. if x[n] = 0 for n<n0, then y[n] = 0 for n<n0).
7. The followings are the impulse responses of LTI systems. Determine whether each system is causal and/or stable. Justify your answers.
   a. h[n] = 2^n .u[3-n]
   b. [1 – 0.99^n].u[n]
   c. 2u[n+5] - u[n] - u[n-5]
   d. h(t) = u(1-t) – (1/2)e^{-t}u(t)
   e. h(t) = e^{15t}[u(t-1) – u(t-100)]
8. Consider an LTI system whose response to the signal $x_1(t)$ is the signal $y_1(t)$ where these signals are defined below. Determine and provide a labeled sketch of the response to the input $x_2(t)$, which is also defined below.

   $x_1(t) = u(t+1) – u(t-1)$,

$$y_1(t) = \begin{cases} t+2, & -2 < t < 0 \\ -t+2, & 0 < t < 2 \end{cases}$$

$$y_2(t) = \begin{cases} t+2 & -2 < t < -1 \\ 1 & -1 < t < 1 \\ -t+2 & 1 < t < 2 \end{cases}$$

9. The input x(t) to a system yields the output y(t), as shown below. Sketch the response y(t) when the input is x(t) = u(t+1) – u(t-2).