Signals and Systems February 29, 2012
Sharif University of Technology CE 40-242
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Date Due: 23/12/90

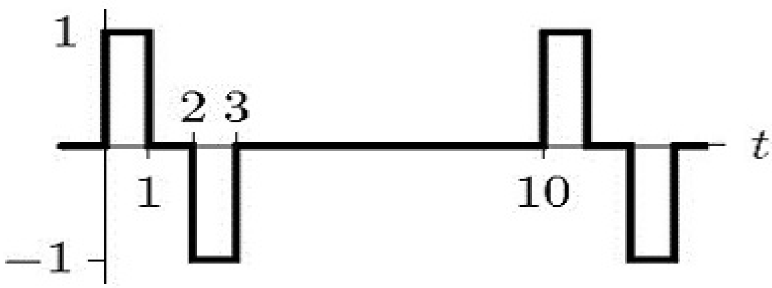
Homework 3 (Chapter 3 & VS[1])

Problems

1. Determine the fundamental frequency, fundamental period, and Fourier sesries coefficients for the following signals:

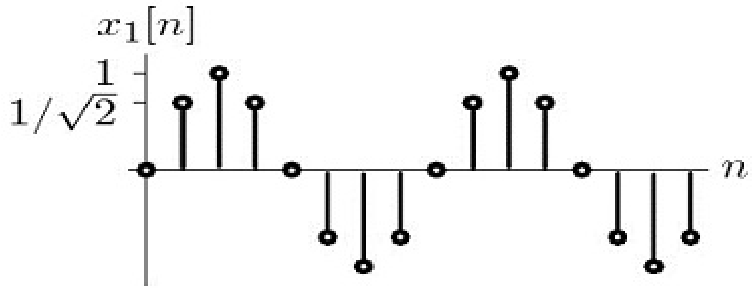
   a. Signals depicted in parts d and f of Figure P3.22 of textbook p. 256

   b.

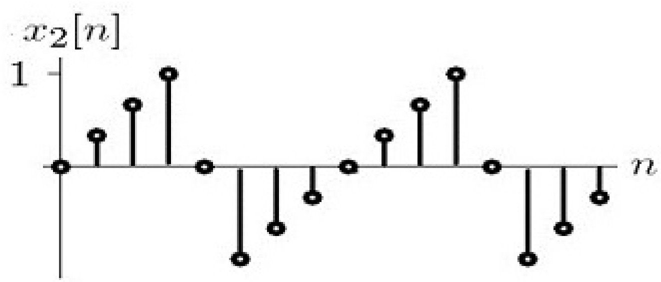


   c. Signals depicted in parts b of Figure P3.28 of textbook p. 258

   d.



   e.



2. Suppose we are given the following information about a signal $x(t)$:

[1] *Vector Space*

(a) $x(t)$ is a real signal.

(b) $x(t)$ is periodic with period $T = 6$ and has Fourier coefficient a_k.

(c) $a_k = 0$ for $k = 0$ and $k > 2$.

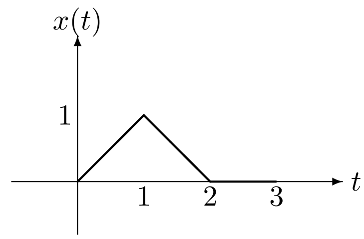
(d) $x(t) = -x(t-3)$

(e) $\frac{1}{6}\int_{-3}^{3} |x(t)|^2 dt = \frac{1}{2}$

(f) a_1 is a positive real number.

Show that $x(t) = Acos(Bt + C)$. and determine the values of the constants A, B and C. (P 3.44 p.262)

3. One period of the periodic signal $x(t)$ with period $T = 3$ is depicted below.



a. Find a_0 without calculating a_k.

b. Evaluate the following sums without calculating a_k.

$$\sum_{k=-\infty}^{+\infty} a_k e^{j4\pi k/3}, \sum_{k=-\infty}^{+\infty} a_{k-2}^2, \sum_{k=-\infty}^{+\infty} a_k^2 e^{j8\pi k/3}$$

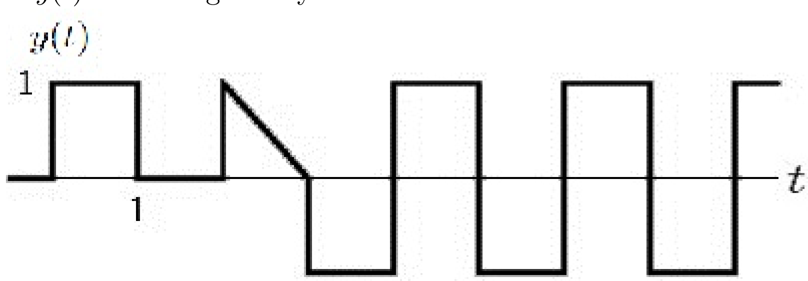
4. Let $x(t) = \frac{u(t)-u(t-2)}{2}$ and $h(t) = e^{-2t}u(t)$,

a. Compute $y(t) = x(t) * h(t)$.

b. Compute $g(t) = \frac{dx(t)}{dt} * h(t)$.

c. What is the relationship between $g(t)$ and $y(t)$?

5. The response of a causal LTI system to the input x(t) which is given by
$x(t) = \sum_{k=0}^{\infty} \delta(t-k)$
is $y(t)$ which is given by



where $y(t)$ is 0 for $t < 0$ and $y(t) = y(t-2)$ for $t > 6$. Sketch the impulse response $h(t)$ of the system.

6. Express the Fourier series coefficients of the following functions in term of Fourier series Coefficients a_k of $x(t)$ when period of $x(t)$ is T.

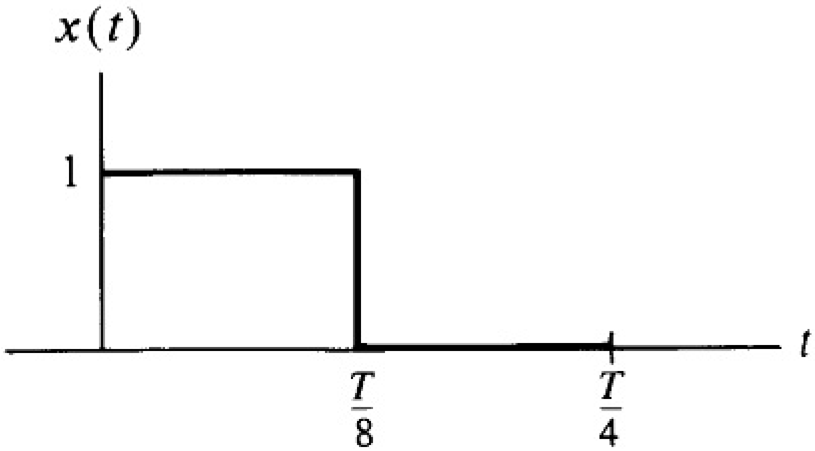
a. $x(t)sin(2k\pi t/T)$

b. $x(-t)$

c. $\mathcal{R}e\{x(t)\}$

d. $\frac{d^2x(t)}{dt^2}$

7. Suppose $x(t)$ is periodic with period T and is specified in the interval $0 < t < T/4$ as shown in the figure



Sketch $x(t)$ in the interval $0 < t < T$ if

(a) the Fourier series has only odd harmonics and $x(t)$ is an even function;

(b) the Fourier series has only odd harmonics and $x(t)$ is an odd function.

8. Consider a linear, time-invariant system with impulse response

$$h[n] = (\tfrac{1}{2})^2$$

Find the Fourier series representation of the output 9[n] for each of the following inputs.

(a) $x[n] = sin(\frac{3\pi n}{4})$

(b) $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$

(c) $x[n] = j^n + (-1)^n$

9. Let $x[n]$ be a periodic sequence with period N and Fourier series representation

$$x[n] = \sum_{<N>} a_k e^{jk(2\pi/N)n},$$

The Fourier series coefficients of each of the following signals can be expressed in terms of a_k. Derive the expressions.(P 3.48 p.265)

a. $x[n-n_0]$

b. $x[n]-x[n-1]$

c. $x[n]-x[n-\frac{N}{2}]$ (assume that N is even)

d. $x[n]-x[n+\frac{N}{2}]$ (assume that N is even; note that this signal is periodic with period $N/2$)

e. $x^*[-n]$

f. $(-1)^n x[n]$ (assume that N is even)

g. $(-1)^n x[n]$ (assume that N is odd; note that this signal is periodic with period $2N$)

h. $y[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

10. Let $B = \{b_1, b_2, ..., b_m\}$ be a subset of R_n which are pariwise orthogonal and none of them is zero. prove that they are linearly independent.

11. If E and $\tilde{E}$ are a pair of biorthogonal bases of a Hilbert space H, prove that for any vector y in H the following equality holds.

$$\|y\|_2^2 = \sum_i < e_i, y >^* < \tilde{e}_i, y >$$

12. Let V be a subspace of Hilbert space H. A projector P_V on V is a linear operator that satisfies:

    (a) $\forall f \in H \rightarrow P_V f \in V$

    (b) $\forall f \in V \rightarrow P_V f = f$

    The projector P_V is orthogonal if $\forall f \in H, \forall g \in V \rightarrow < f - P_V f, g >= 0$. If P_V is an orthogonal projector on V prove that:

    (a) $\forall f \in H \rightarrow \|f - P_V f\| = min_{g \in V} \|f - g\|$

    (b) if $\{e_n\}_n \in N$ is an orthogonal basis of V, then $P_V\ f = \sum_{n=0}^{+\infty} \frac{<f, e_n>}{\|e_n\|^2} e_n$