Signals and Systems March 13, 2012
Sharif University of Technology CE 40-242
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Date Due:

Homework 4 (Chapter 4)

Problems

1. Compute the Fourier transform of each of the following signals:

   a. $x(t) = e^{-3|t|} \sin 2t$

   b. $[te^{-2t} \sin 4t]u(t)$

   c. $x(t) = \begin{cases} b^2(1 - \frac{|t|}{2a}), & |t| < 2a \\ 0, & o.w. \end{cases}$, Where a, $b > 0$.

   d. $[\frac{\sin \pi t}{\pi t}][\frac{\sin 2\pi(t-1)}{\pi(t-1)}]$

   e. $x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

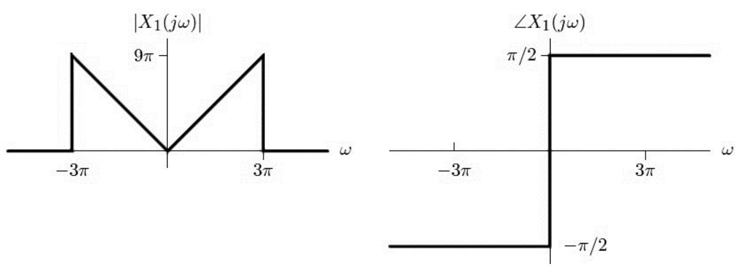
   f. $x(t)$ as shown in Figure P4.21(b) in p. 338 of textbook.

2. Determine the continuous-time signal corresponding to each of the following transforms:

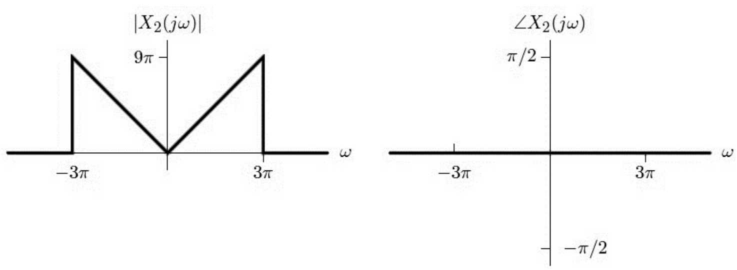
   a. $X(j\omega) = j[\delta(\omega + 1) - \delta(\omega - 1)] - 3[\delta(\omega - \pi) + \delta(\omega + \pi)]$

   b. $X(j\omega) = \frac{2 \sin[3(\omega - 2\pi)]}{\omega - 2\pi}$

   c.



   d.



3. Determine which, if any, of the real signals depicated in below have Fourier transforms that satisfy each of the following conditions:

   1. $\mathcal{R}e\{X(j\omega)\} = 0$

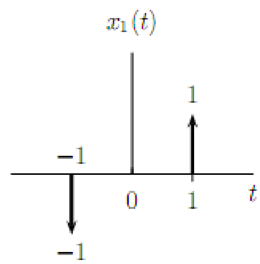
2. $\mathcal{I}m\{X(j\omega)\} = 0$

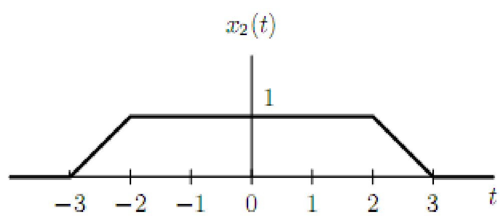
3. There exists a real α such that $e^{j\alpha\omega}X(j\omega)$ is real

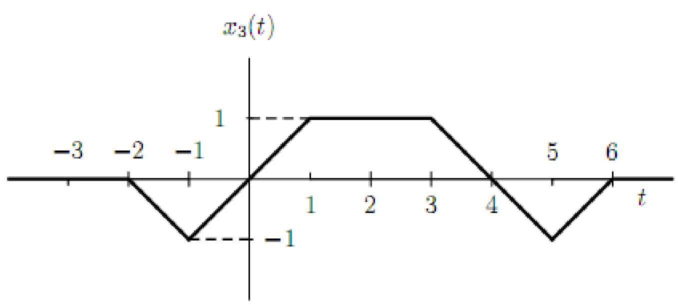
4. $\int_{-\infty}^{\infty} X(j\omega)d\omega = 0$

5. $\int_{-\infty}^{\infty} \omega X(j\omega)d\omega = 0$

6. $X(j\omega)$ is periodic



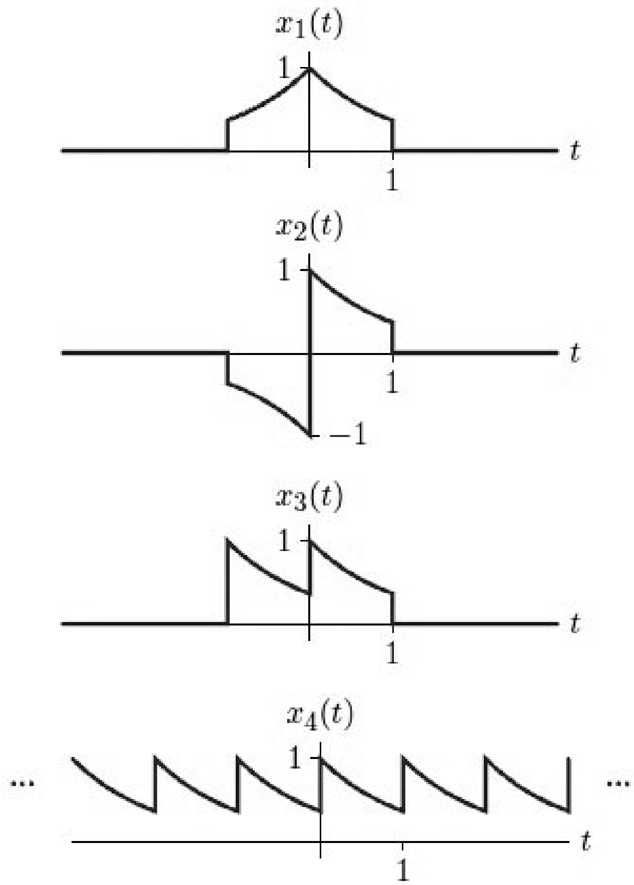




4. Let $X(j\omega)$ represent the Fourier transform of

$$x(t) = \begin{cases} e^{-t} & 0 < t < 1 \\ 0 & o.w \end{cases}$$

Express the Fourier Transforms of each of the following signals in terms of $X(j\omega)$.



5. Suppose an LTI system is described by the following LCCDE:

$$\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + 3y(t) = \frac{4dx(t)}{dt} - x(t)$$

   Show that $Y(j\omega)$ can be expressed as $Y(j\omega) = H(j\omega)X(j\omega)$ and find $H(j\omega)$.

6. Let $x_1(t)$ represent the input to an LTI system, where

$$x_1(t) = \sum_{k=-\infty}^{+\infty} \alpha^{|k|} e^{jk\frac{\pi}{4}t}$$

   for $0 < \alpha < 1$. The frequency response of the system is

$$H(j\omega) = \begin{cases} 1 & |\omega| < W \\ 0 & o.w \end{cases}$$

   What is the minimum value of W so that the average energy in the output signal will be at least 90% of that in the input signal.

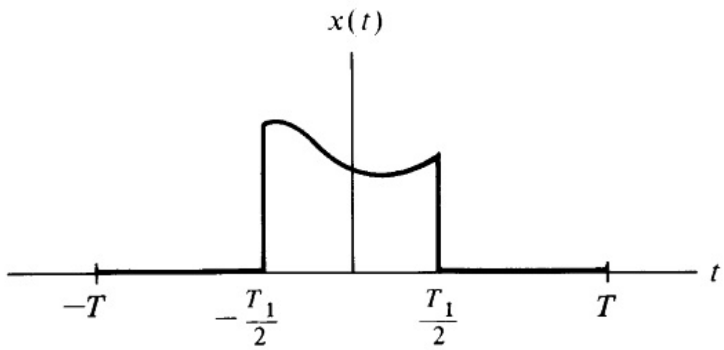
7. Consider the impulse train

$$p(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT)$$

(a) Find the Fourier series of $p(t)$.

(b) Find the Fourier transform of $p(t)$.

(c) Consider the signal $x(t)$ shown in following figure, where $T_1 < T$.

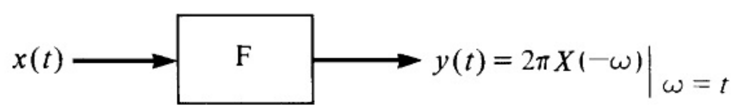


Show that the periodic signal $\tilde{x}(t)$, formed by periodically repeating $x(t)$, satisfies

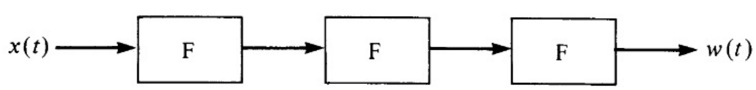
$$\tilde{x}(t) = x(t) * p(t)$$

(d) find the Fourier transform of $\tilde{x}(t)$ in terms of the Fourier transform of $x(t)$.

8. Suppose that the system F takes the Fourier transform of the input, as shown:



What is $w(t)$ calculated in following figure:



9. (a) Show that the three LTI systems with impulse responses

$$h_1(t) = u(t),$$

$$h_2(t) = -2\delta(t) + 5e^{-2t}u(t),$$

$$h_3(t) = 2te^{-t}u(t)$$

all have the same response to $x(t) = \cos(t)$.

(b) Find the impulse response of another LTI system with the same response to $\cos(t)$.

This problem illustrates the fact that the response to $\cos(t)$ cannot be used to specify an LTI system uniquely.