Signals and Systems
Sharif University of Technology
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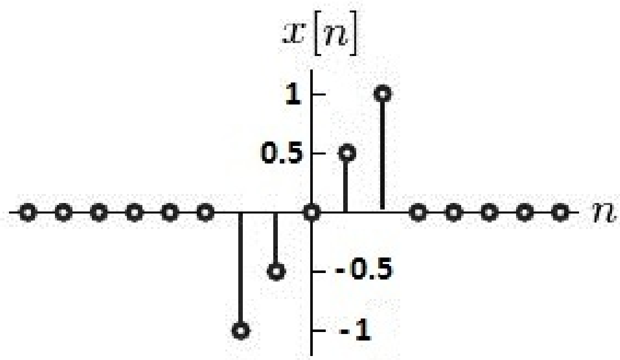
CE 40-242

Date Due: Ordibehesht 5, 1391

Homework 5 (Chapter 5)

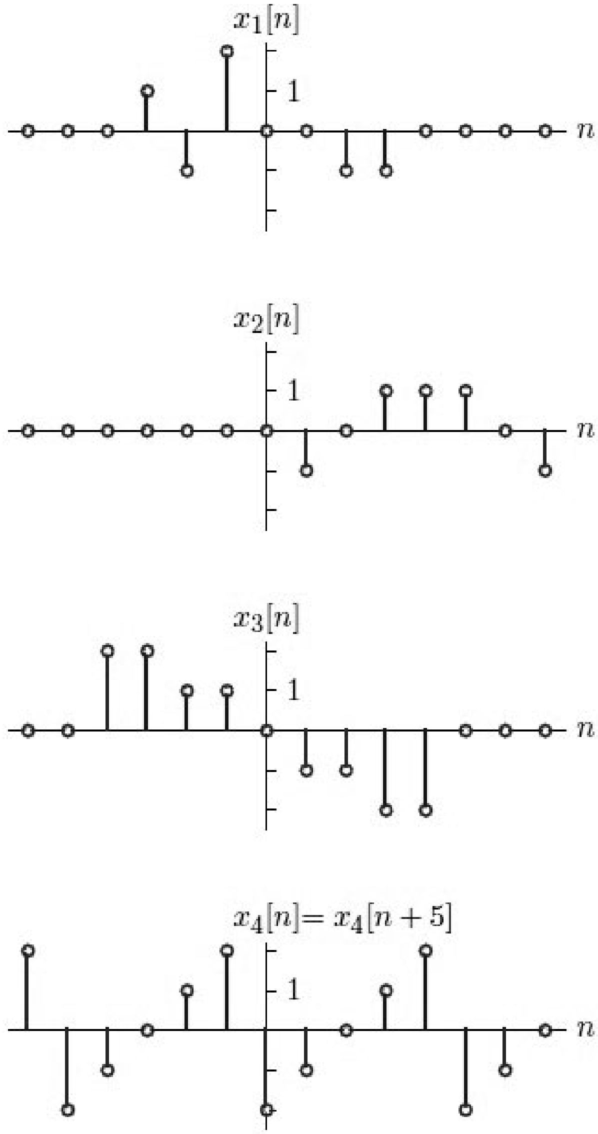
Problems

1. Computing the Fourier transform.
   - 5-21. a
   - 5.21. b
   - 5.21. e
   - 5.21. h
   - 5.21. j
   -



2. Determining corresponding signals of the transforms.
   - 5-22. a
   - 5-22. b
   - 5-22. d
   - 5-22. e
   - 5-22. f
   - 5-22. h

3. For each of the DT signals $x_1[n]$ through $x_4[n]$ (below), determine whether the conditions listed are satisfied:
   (a) $X(e^{j0}) = 0$
   (b) $\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega = 0$
   (c) $X(e^{j\omega})$ is Purely Imaginary.



4. A particular LTI system is described by the difference equation:

$$y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] - x[n-1]$$

   (a) Find the impulse response of the system.
   (b) Evaluate the magnitude and phase of the system frequency response at $\omega = 0, \omega = \pi/4, \omega = -\pi/4$ and $\omega = 9\pi/4$.

5. Consider a system consisting of the cascade of two LTI systems described by the difference equation:

$$y[n] = x[n] - x[n-1] + 3x[n-4]$$

and

$$y[n] - \frac{1}{3}y[n-1] = x[n]$$

(a) Find the difference equation describing the overall system.
(b) Determine the impulse response of the overall system.
(c) If g[n] is response of system to input x[n], find system response to 2x[n-1] in term of g[n].

6. if $x[n]$ and $X[e^{j\omega}]$ denote a sequence and its Fourier transform, determine in terms of x[n] the sequence corresponding to:

   (a) $X(e^{j(\omega-\omega_0)})$
   (b) $Re\{X(e^{j\omega})\}$
   (c) $Im\{X(e^{j\omega})\}$
   (d) $|X(e^{j\omega})|^2$
   Hint: Write your answer in terms of a convolution.

7. Let $X(e^{j\omega})$ be the Fourier transform of $x[n]$. Derive expressions in terms of $X(e^{j\omega})$ for the Fourier transforms of the following signals. (Do NOT assume that x[n] is real.)
   [Problem 5.37. page 411]

   (a) $Re\{x[n]\}$
   (b) $X^*[-n]$
   (c) $Ev\{x[n]\}$

8. Suppose the input to system L is:

$$x(t) = \frac{1}{1+t^2}$$

   and system L has an impulse response whose Fourier transform is:

$$H(\omega) = \begin{cases} 1 & -1 \leq \omega < 1 \\ 0 & o.w. \end{cases}$$

   If the output of the system is y(t), find the energy of y(t),

$$E_y = \int_{-\infty}^{\infty} y^2(t)dt$$

   and express E_y as a percentage of the input's energy (E_x)

$$E_x = \int_{-\infty}^{\infty} x^2(t)dt$$

9. Compute the N-point DFT of each of the following sequences:

   (a) $x[n] = \delta[n]$
   (b) $x[n] = \delta[n-n_0]$, Where $0 < n < N$
   (c) $x[n] = a^n$, Where $0 < n < N$
   (d) $x[n] = u[n] - u[n-n_0]$, Where $0 < n_0 < N$
   (e) $x[n] = 4 + cos^2(\frac{2\pi n}{N})$, $n = 0, 1, ..., N-1$

10. If $x_1[n]$ and $x_2[n]$ are N-point sequences with N-point DFTs $X_1(k)$ and $X_2(k)$, respectively,

(a) show that:

$$\sum_{n=0}^{N-1} x_1[n].x_2^*[n] = \frac{1}{N}\sum_{k=0}^{N-1} X_1(k).X_2^*(k)$$

(b) Evaluate the sum:

$$S = \sum_{n=0}^{N-1} x_1[n].x_2^*[n]$$

When:

$$x_1[n] = cos(\frac{2\pi n k_1}{N}), x_2[n] = cos(\frac{2\pi n k_2}{N}), n = 0, 1, ..., N-1$$