Signals and Systems
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CE 40-242

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Homework 6 (Chapter 7)

Problems

1. Let $x(t)$ be a signal with Nyquist rate ω_0. Determine the Nyquist rate for each of the following signals:

   a. $ax(t) + bx(t-2)$

   b. $x(at) + x(bt-2) \forall a, b \in N$

   c. $\frac{dx(t)}{dt} + x(t)$

   d. $x^2(t)$

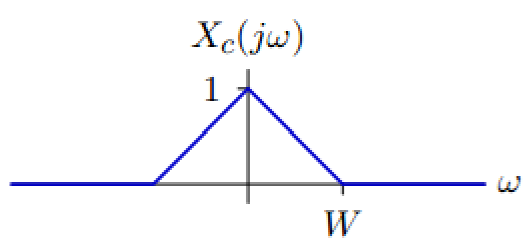
   e. $x^3(t)$

   f. $x(t)sin(\omega_0 t)$

2. A continiouse time signal x(t) is converted to a discrete time signal $x_d[n]$ as follows:

$$x_d[n] = \begin{cases} x_c(nT) & \text{n is odd} \\ -x_c(nT) & \text{n is even} \end{cases}$$

   a. Assume that the Fourier transform of $x_c(t)$ is $X_c(j\omega)$ shown below.



   b. Assume that $x_c(t)$ is bandlimitted to $-W \leq \omega \leq W$. Determine the maximum value of W for which the original signal $x_c(t)$ can be reconstructed from samples $x_d[n]$.

   c. Draw the component diagram of the system that reconstructs $x_c(t)$ from $x_d[n]$.

3. Given the following frequency response for a CT filter:

$$H(j\omega) = \frac{1}{(a+j\omega)^2}$$

   Draw the diagram of the system that uses a core discrete time filter that has the same output $y_c(t)$ as the mentioned CT filter. Find $h_d[n]$ of the core discrete time filter.

4. 7.41 P577

5. 7.34 P568 use $5\pi/13 < |\omega| < \pi$ instead of the original question inequality.

6. 7.30 P567 use $\frac{d^2 y_c(t)}{dt} + y_c(t-1) = x_c(t)$ instead of the question differential equation.

7. Suppose there is a lossy channel that just passes signals with frequency $A \leq \omega \leq B$. In other words the frequency response of the channel is 1 in $\omega \in (A, B)$ and unknown for other frequencies. TV stations want to send their signals over this channel. It is known that the stations produces signals in frequency range $\omega \in (C, D)$. Design the sender adaptor and reciver adaptor for the TV stations. Assume $A, B, C, D > 0$. Suppose you have to use all of the frequency range (A, B). Hint: use C/D and D/C convertors.

8. On a full duplex connection both ends can transmit band limitted signals simultaneously in both directions. Suppose devices A and B produce discrete time signals x[n] and the cable connecting these two can transmit signals in range $(5\pi, 10\pi)$. Design the sender and reciver adaptor of each device in a way that they can communicate simultaneously through the channel using their adaptors.
   You should design four different adaptors, a sender and a reciever for each device using the following diagram. You must consider an unused frequency range of length π to avoid frequency cofilicts due to noise.
   (Hint: device A would know that it should recive signals on some specific frequencie range that sender B uses for sending its signals, so you need two instances of the following diagram and you have to define two frequency ranges in the given interval)