Date Due: Esfand 13^{th}, 1391

Homework 3 (Chapter 3)

Problems

1. Determine the Fourier series coefficients for the following signal:

$$x(t) = 2 + 4cos(w_0 t) + 2sin(\frac{5}{2}w_0 t + \frac{\pi}{4}) + cosh(3w_0 t)$$

2. If the series $A + \sum_{n=1}^{\infty} a_n cos(\frac{n\pi x}{L}) + b_n sin(\frac{n\pi x}{L})$ converges uniformly to $f(x)$ in $(-L, L)$, show that for $n = 1, 2, 3, ...$

   a. $a_n = \frac{1}{L}\int_{-L}^{L} f(x)cos(\frac{n\pi x}{L})dx$

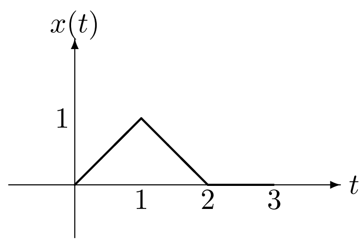
   b. $b_n = \frac{1}{L}\int_{-L}^{L} f(x)sin(\frac{n\pi x}{L})dx$

   c. $A = \frac{a_0}{2}$

3. Expand $f(x) = x^2, 0 < x < 2\pi$ in a Fourier series with period$= 2\pi$

4. Using the results of problem 3 prove that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ... = \frac{\pi^2}{6}$

5. One period of the periodic signal $x(t)$ with period $T = 3$ is depicted below.



   a. Find a_0 without calculating a_k.

   b. Evaluate the following sums without calculating a_k.

$$\sum_{k=-\infty}^{+\infty} a_k e^{j4\pi k/3}, \sum_{k=-\infty}^{+\infty} |a_{k-2}|^2, \sum_{k=-\infty}^{+\infty} a_k^2 e^{j8\pi k/3}$$

6. In each of the following, we specify the Fourier series coefficients of a continuous-time signal that is periodic with period 4. Determine the signal $x(t)$ in each case.

   (a) $a_k = \begin{cases} 0 & k = 0 \\ j^k \frac{sin(k\pi/4)}{k\pi}, & otherwise \end{cases}$

   (b) $a_k = (-1)^k \frac{sin(k\pi/8)}{2k\pi}$

   (c) $a_k = \begin{cases} jk & |k| < 3 \\ 0 & otherwise \end{cases}$

(d) $a_k = \begin{cases} 1 & k \quad even \\ 2 & k \quad odd \end{cases}$

7. Let $x[n]$ be a periodic sequence with period N and Fourier series representation

$$x[n] = \sum_{<N>} a_k e^{jk(2\pi/N)n},$$

The Fourier series coefficients of each of the following signals can be expressed in terms of a_k.

a. $x[n-n_0]$

b. $x[n] - x[n-1]$

c. $x[n] - x[n-\frac{N}{2}]$ (assume that N is even)

d. $x[n] + x[n+\frac{N}{2}]$ (assume that N is even; note that this signal is periodic with period $N/2$)

e. $x^*[-n]$

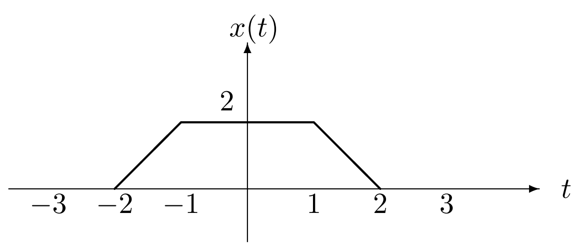
f. $(-1)^n x[n]$ (assume that N is even)

g. $(-1)^n x[n]$ (assume that N is odd; note that this signal is periodic with period $2N$)

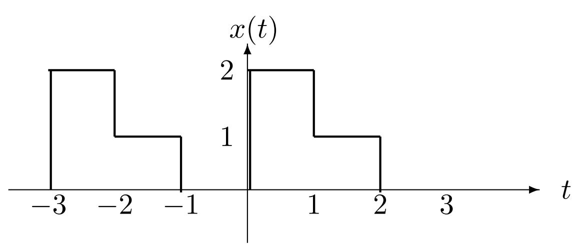
h. $y[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

8. Determine the Fourier series representation for the following signals:

(a) T=6



(b) T=3



(c) $x(t) = e^{-t}$ for $-1 < t < 1$ and $x(t)$ is periodic with period 2.

Practical Assignment

1. Consider the periodic square wave $x(t) = \begin{cases} 1, & |t| \leq 1 \\ 0, & 1 < t < 3 \end{cases}$ defined over one period. Obtain a general expression for its Fourier series coefficients. Calculate the first 100 coefficients in MATLAB. Display the reconstructed signal using the first 1, 3, 7, 19, 41, and 79 coefficient(s) overlaid on the original signal. Calculate the Mean Square Errors (MSEs) between these reconstructed signals in all 100 possible partial summations and the original signal. Plot the MSE versus number of coefficients.

2. Figure 1 shows the sawtooth signal. This signal is defined by $f(x) = \frac{x}{2L}$, $0 \leq x \leq 2L$ during one period of time. The Fourier series of the periodic form of the signal is given by $f(x) = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{\text{inf}} \frac{1}{n} sin(\frac{n\pi x}{L})$. Use the given sample code on the web site of the course and modify it for rectangular function.

   (a) For $L = 1$, plot the first term of the Fourier serires, the first ten terms and first twenty terms of the series. You can demonstrate the Gibbs phenomena for the sawtooth function.

   (b) What is the effect of scaling, time shifting and number of samples on the Gibbs phenomena? Demonstrate it by plotting the scaled and shifted version of $f(x)$.

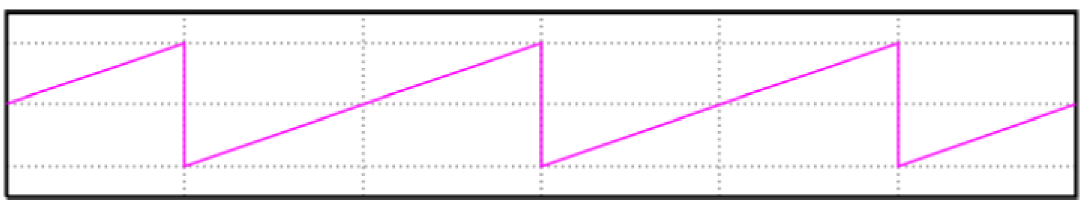


Figure 1. Sawthoot wave