Signals and Systems
Sharif University of Technology
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CE 40-242

Date Due: Esfand 27, 1391

Homework 4 (Chapter 4)

Problems

1. Compute the Fourier transform of each of the following signals:

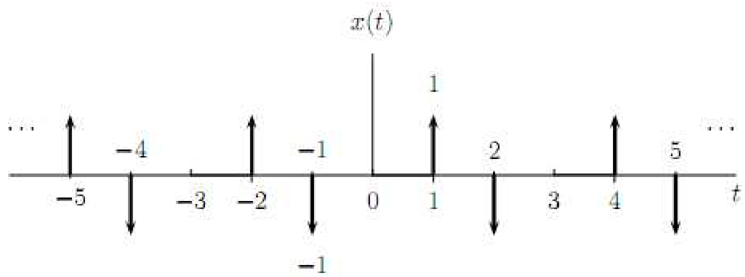
   a. $x(t) = e^{-4|t|}\cos(5\pi t)$

   b. $x(t) = \begin{cases} -\frac{1}{2}, & t \leq -\frac{1}{2} \\ t, & -\frac{1}{2} < t \leq \frac{1}{2} \\ \frac{1}{2}, & \frac{1}{2} < t \end{cases}$

   c. $x(t) = \begin{cases} 2 + t^2, & 0 < t < 1 \\ 0, & otherwise \end{cases}$

   d. $[\frac{\sin \pi t}{\pi t}][\frac{\sin 2\pi(t-1)}{\pi(t-1)}]$

   e. The signal $x(t)$ depicted below:



2. Determine the continuous-time signal corresponding to each of the following transforms:

   a. $X(j\omega) = e^{6j\omega}\dfrac{1}{(3 + j\omega)^2}$

   b. $X(j\omega) = \dfrac{1}{2 + 3j\omega - \omega^2}$

   c. $X(j\omega) = \dfrac{\sin^2(3\omega)\cos\omega}{\omega^2}$

3. Assume that $x(t)$ is purely imaginary and the Fourier transform of $x(t)$ is $X(j\omega)$

   a. Prove $X^*(j\omega) = -X(-j\omega)$

   b. Determine whether the corresponding time-domain signal $X(j\omega)$ is (i) real, imaginary or neither and (ii) even or odd or neither. Do this without evaluating the inverse Fourier transform of the given transform.

$$X(j\omega) = \frac{\sin 2\omega}{\omega} e^{j(2\omega - \frac{\pi}{2})}$$

4. Determine which, if any, of the real signals depicated in below have Fourier transforms that satisfy each of the following conditions:

   1. $\mathcal{R}e\{X(j\omega)\} = 0$

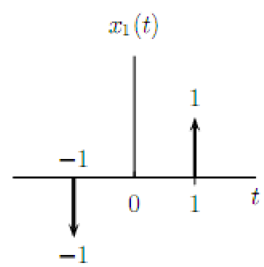
   2. $\mathcal{I}m\{X(j\omega)\} = 0$

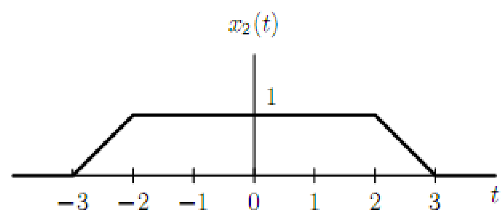
   3. There exists a real α such that $e^{j\alpha\omega}X(j\omega)$ is real

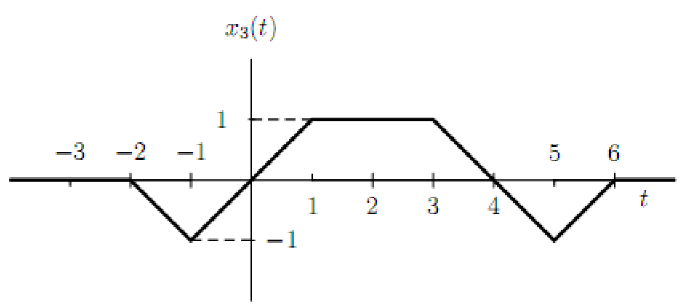
   4. $\int_{-\infty}^{\infty} X(j\omega)d\omega = 0$

   5. $\int_{-\infty}^{\infty} \omega X(j\omega)d\omega = 0$

   6. $X(j\omega)$ is periodic







5. Suppose $g(t) = x(t)\cos t$ and the Fourier transform of the $g(t)$ is

$$G(j\omega) = \begin{cases} 1, & |\omega| \leq 2 \\ 0, & otherwise \end{cases}$$

   a. Determine $x(t)$.

   b. Specify the Fourier transform $X_1(j\omega)$ of a signal $x_1(t)$ such that

$$g(t) = x_1(t)\cos(\frac{2}{3}t)$$

6. The input and the output of a causal LTI system are related by the differential equation

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

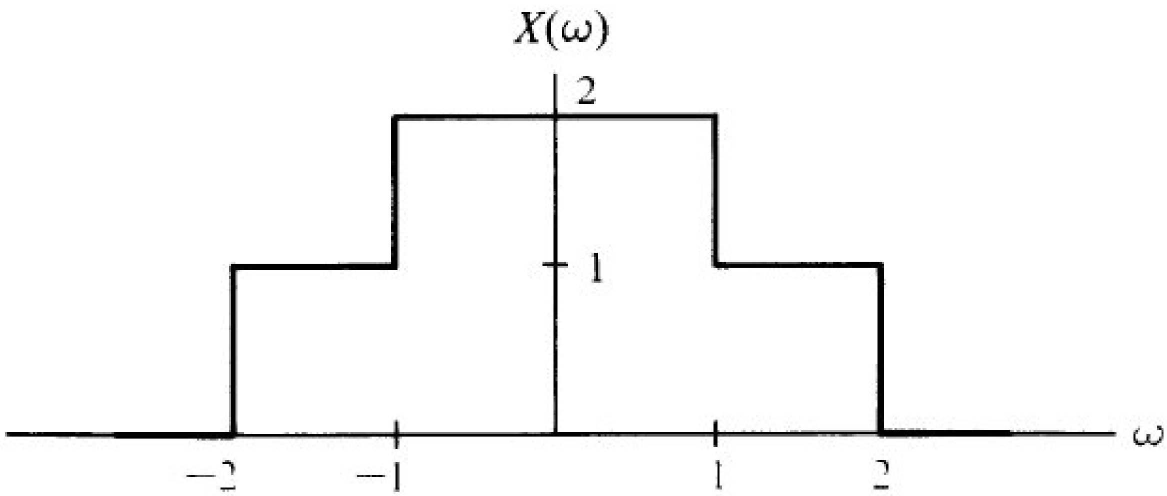
a. Find the impulse response of this system.

b. What is the response of this system if $x(t) = te^{-2t}u(t)$?

c. Repeat part (a) for the causal LTI system described by the equation

$$\frac{d^2y(t)}{dt^2} + \sqrt{2}\frac{dy(t)}{dt} + y(t) = 2\frac{d^2x(t)}{dt^2} - 2x(t)$$

7. **a.** Determine the energy in the signal $x(t)$ for which the Fourier transform $X(j\omega)$ is depicted below.



b. Find the inverse Fourier transform of $X(j\omega)$ of part (a).

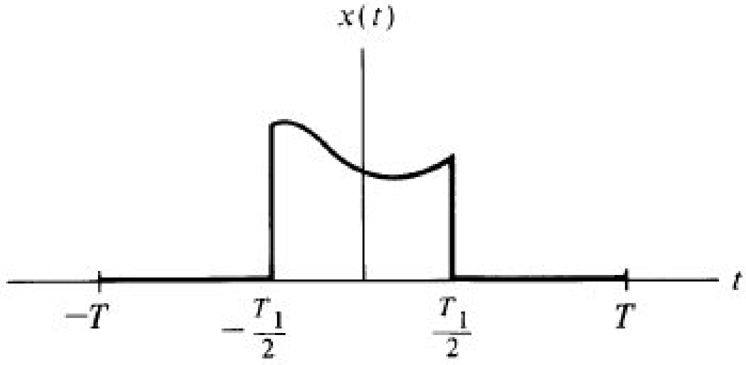
8. Consider the impulse train

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

a. Find the Fourier series of $p(t)$.

b. Find the Fourier transform of $p(t)$.

c. Consider the signal $x(t)$ that depicted below where $T_1 < T$.



show that the periodic signal $\bar{x}(t)$, formed by periodically repeating $x(t)$, satisfies.

$$\bar{x} = x(t) * p(t)$$