*Signals and Systems*
Sharif University of Technology
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CE 40-242

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Date Due: Ordibehesh 2nd, 1392

# Homework 5
# (Chapter 5)

**Problems**

1. Compute the Fourier transform of the following signals.

   - $x[n] = \frac{e^{j\frac{\pi}{5}n} sin(\frac{2\pi}{5}n)}{\pi n}$
   - $x[n] = u[-n-1] - u[n+3]$
   - $x[n] = cos(\frac{5\pi}{3}n) - sin(\frac{7\pi}{3}n)$
   - $x[n] = (\frac{1}{3})^n u[n] cos(\frac{5\pi}{8}n)$
   - $x[n] = (n+3)(\frac{1}{4})^{|n|}$

2. Determining corresponding signals of the transforms.

   - $X(e^{j\omega}) = 6 - e^{j\omega} + 8e^{-j3\omega} - 16e^{-j11\omega} + 4e^{j4\omega}$
   - $X(e^{j\omega}) = sin^2(3\omega) + cos^2(2\omega)$
   - $X(e^{j\omega}) = \frac{1+3e^{-j3\omega}}{1+\frac{1}{4}e^{-j\omega}}$
   - $X(e^{j\omega}) = \frac{e^{-j2\omega} sin(\frac{5}{2}\omega)}{sin(\frac{\omega}{2})}$
   - $X(e^{j\omega}) = \begin{cases} 1, & 0 \leq |\omega| < \frac{\pi}{4}, \quad \frac{\pi}{2} < |\omega| \leq \pi \\ 0, & \frac{\pi}{4} < |\omega| < \frac{\pi}{2} \end{cases}$

3. Consider an LTI system described by the following difference equation:

$$y[n] + \frac{1}{2}y[n-1] = x[n]$$

   (a) Find the impulse response of the system.

   (b) Is this system causal? Why?

   (c) Is the inverse of this system causal? Why?

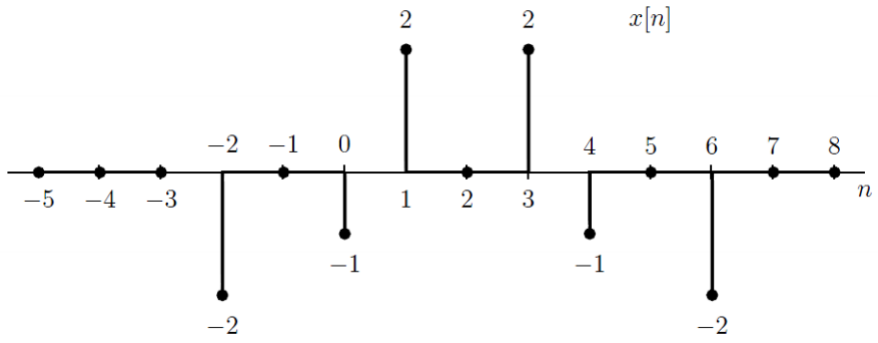
   (d) Is this system a lowpass, highpass or bandpass filter? (Hint: You can sketch the magnitude of the frequency response.)

4. Let $X(e^{j\omega})$ denote the Fourier transform of the signal $x[n]$ depicted below.

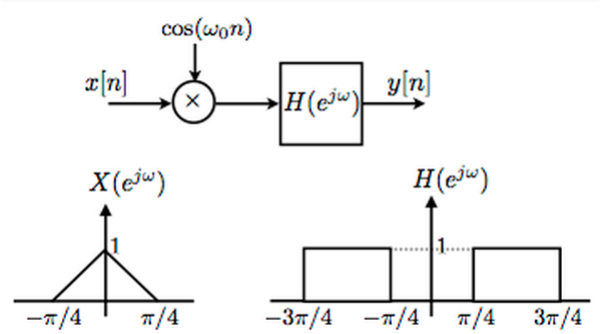
   (a) Find $a$ such that $e^{ja\omega}X(e^{j\omega})$ is real.

   (b) Find $X(e^{j\pi})$.

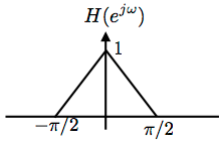
   (c) Find and sketch the signal whose Fourier transform is $Real\{X(e^{j\omega})\}$.



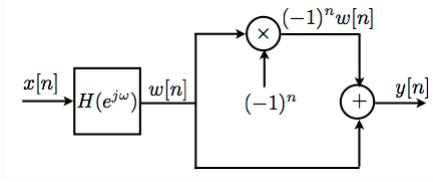

5. Consider the following system with the input $x[n]$ and the bandpass filter $h[n]$. Determine the value of $\omega_0$ that maximaizes the energy of the output for the given $X(e^{j\omega})$ and $H(e^{j\omega})$.




6. Consider a lowpass filter with the following frequency response. What is the output $y[n]$ when the input to this filter is $x[n] = cos(\frac{\pi}{5}n) + sin(\frac{\pi}{4}n) + \frac{1}{2}cos(\frac{3\pi}{4}n)$?




7. Consider the following system. What is the value of $\sum_{n=-\infty}^{\infty} y[n]$ when the input is $x[n] = \delta[n]$ and $H(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{\pi}{2} \\ 0, & else \end{cases}$ ?



**Practical Problems**

1. Write a MATLAB function to compute the DTFT of a finite-duration sequence. The format of the function should be

   ```
   function [X] = dtft(x, n, w)
   % [X] = dtft(x, n, w)
   % X = DTFT values computed at w frequencies
   % x = finite duration sequence over n
   % n = sample position vector
   % w = frequency location vector
   ```

2. Write a MATLAB code to compute the DFTF of the signal $x[n] = \begin{cases} 1, & |n| \leq N_1 \\ 0, & |n| > N_1 \end{cases}$. Plot $X(e^{j\omega})$ and its magnitude for $N_1 = 1, 2, 4$. (You can use 'subplot' to have all the results in one figure.)