Signals and Systems April 23, 2013
Sharif University of Technology CE 40-242
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Date Due: Ordibehesht 30, 1392

Homework 7(Chapters 9&10)

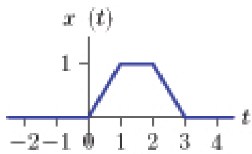
Problems:

1. Determine the Laplace transforms (including the regions of convergence) of each of the following signals.

   a. $x(t) = e^{-2(t-3)}u(t-3)$

   b. $x(t) = (1-(1-t)e^{-3t})u(t)$

   c. $x(t) = |t|e^{-|t|}$

   d. 

2. Determine all possible signals with Laplace transforms of the following forms. For each signal, indicate the associated region of convergence.

   a. $X(s) = \frac{s+2}{(s+1)^2}$

   b. $X(s) = \frac{1}{s^2(s+1)}$

   c. $X(s) = \frac{s+1}{s^2+2s+2}$

   d. $X(s) = (\frac{1-e^{-s}}{s})^2$

3. A system is represented by the following differential equation:

   $\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + 5y(t) = \frac{d^2x(t)}{dt^2} - 2\frac{dx(t)}{dt} + x(t)$

   a. Find the system function H(s) of the continuous-time LTI system.

   b. Does this system have a stable and causal inverse? Why or why not?

4. Use the initial and final value theorems (where applicable) to find $x(0)$ and $x(\infty)$ for the signals with the following Laplace transforms:

   a. $(\frac{1}{s})e^{-sT}$

   b. $\frac{1}{s(s+1)^2}$

   c. $\frac{1}{s^2(s+1)}$

   d. $\frac{1}{s^2+1}$

   e. $\frac{(s+1)^2-1}{((s+1)^2+1)^2}$

   f. $\frac{1-e^{-sT}}{s}$

5. We are given the following fact about a real signal $x(t)$ with Laplace transform $X(s)$

   a. $X(s)$ has exactly two poles.

   b. $X(s)$ has no zeros in the finite s-plane.

   c. $X(s)$ has a pole at $s = -1 + j$.

   d. $e^{2t}x(t)$ is not absolutely integrable.

   e. $X(0) = 8$.

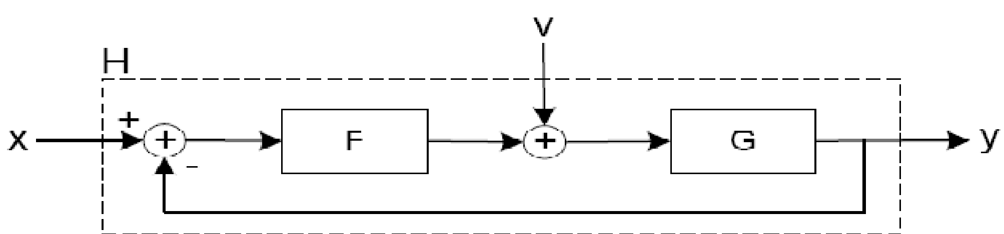
   Determine $X(s)$ and specify its region of convergence.

6. Let $H(s)$ represent the system function for a causal, stable system. The input to the system consists of the sum of three terms, one of which is an impulse $\delta(t)$ and another complex exponential of the form $e^(s_0 t)$, where s_0 is a complex constant. The output is:

   $y(t) = -6e^{-t}u(t) + \frac{4}{34}e^{4t}cos(3t) + \frac{18}{34}e^{4t}sin(3t) + \delta(t)$

   Determine H(s),consistently with this information. Explain your answer.

7. Consider a feedback interconnection of two causal, continuous-time LTI systems, as shown in the block diagram below.



$$F(s) = \frac{K}{s}$$
$$G(s) = \frac{s+2}{s-1}$$

   a. Determine if any of F or G are stable.

   b. Determine all values of K for which the closed-loop system is stable.

8. A right-sided sequence x[n] has z-transform $X(z)$ given by

$$X(z) = \frac{3z^{-10} + z^{-7} - 5z^{-2} + 4z^{-1} + 1}{z^{-10} - 5z^{-7} + z^{-3}}$$

   Determine $x[n]$ for $n < 0$.

9. Consider a sequence $x_1[n]$ with z-transform $X_1(z)$ and a sequence $x_2[n]$ with z-transform $X_2(z)$, where $x_1[n]$ and $x_2[n]$ are related by $x_2[n] = x_1[-n]$.
show that $X_2(z) = X_1(\frac{1}{z})$ and, from this, show that, if $X_1(z)$ has a pole (or zero) at $z = z_0$ then $X_2(z)$ has a pole (or zero) at $z = 1/z_0$

10. we are given the following five facts about a discrete-time signal $x[n]$ with Z-transform $X(z)$

    a. $x[n]$ is real and right-sided.
    b. $X(z)$ has exactly two poles.
    c. $X(z)$ has to zeros at the origin.
    d. $X(z)$ has poles at $z = 0.5e^{j\frac{\pi}{3}}$
    e. $X(1) = \frac{8}{3}$

    Determine $X(z)$ and specify its region of convergence.

11. A casual LTI system is described by the difference equation:

$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

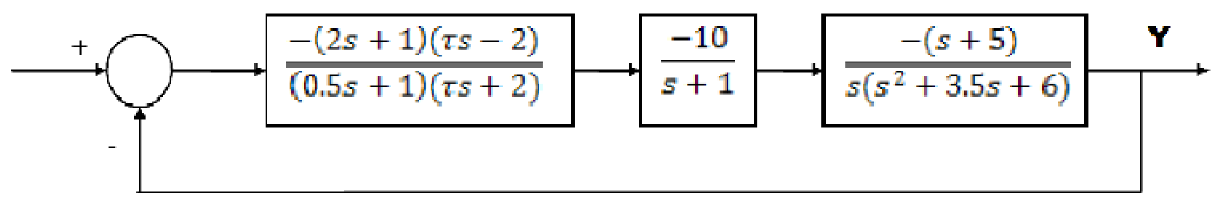
    a. Find the system function $H(z) = (Y(z))/(X(z))$ for this system. Plot the poles and zeros of $H(z)$ and indicate the region of convergence.
    b. Find the unit sample response of the system.(You should have found this to be an unstable system. Find a stable (non casual) unit sample response that satisfies the difference equation.)

12. Determine the Z transform (including the region of convergence) for each of the following signals:

    a. $x_1[n] = (\frac{1}{2})^n u[n-3]$
    b. $x_2[n] = (1+n)(\frac{1}{3})^n u[n]$
    c. $x_3[n] = n(\frac{1}{2})^{|n|}$

Practical Assignment:

I. Consider the following feedback system, in which the output is Y.



a. If τ in this system becomes 0.25, then how long it takes for the output Y to reach the 90% of its final value. Is the system stable in this manner? sketch Pole zero map and step response of the system.
b. Answer part (a) if τ becomes 0.5.
c. Determine the maximum value of τ for which the system remains stable.

⋆ **Notice:** For Practical Assignment you have to include your documentation.